INDIAN INSTITUTE OF SCIENCE

Water Resources Systems:
Modeling Techniques and Analysis

Lecture - 19

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Summary of the previous lecture

- Simulation
  - Reservoir operating policy
  - Multi reservoir simulations
  - Simulation of real time reservoir operation

- Multi-objective optimization

Multi-objective Planning

Weighting method:

- Attach weights to each objective

Max $Z = w_1 Z_1 + w_2 Z_2 + \ldots\ldots\ldots\ldots + w_p Z_p$

s.t.

$g_i(X) \leq b_i$ $\qquad i = 1, 2, \ldots\ldots, m$

where w_j is relative weight (non-negative)

Multi-objective Planning

- The weights reflect the trade-off of pairs of objective functions.
- These weights are varied systematically and the model is solved for each set to generate a set of technically efficient solutions.
- By varying the weights in each case, a wide range of plans are obtained for further analysis before the best one is selected.

Multi-objective Planning

Constraint method:

- One objective is maximized with lower bounds on all the others.

Max $Z_j(X)$

s.t.

$g_i(X) \leq b_i \qquad i = 1, 2, \ldots\ldots, m$

and

$Z_k(X) \geq L_k \qquad \forall \quad k \neq j$

Lower bounds

Multi-objective Planning

- Any set of feasible values of L_k resulting in a solution with binding constraints gives an effective alternative.
- If the constrained method of formulation can be solved using LP, it is particularly useful to conduct sensitivity analysis to infer the implied tradeoffs for given right-hand side values of the binding constraints.
- The dual variables of the binding constraints with L_k on the right-hand side are the marginal rates of transformation of the objectives $Z_j(X)$ and $Z_k(X)$.

Example – 1

A reservoir is planned both for gravity and lift irrigation through withdrawals from its storage. If X_1 is the allocation of water to gravity irrigation and X_2 is the allocation for lift irrigation, two objectives are planned to be maximized and are expressed as

Max $Z_1(X) = 5X_1 - 4X_2$ Max $Z_2(X) = -2X_1 + 8X_2$

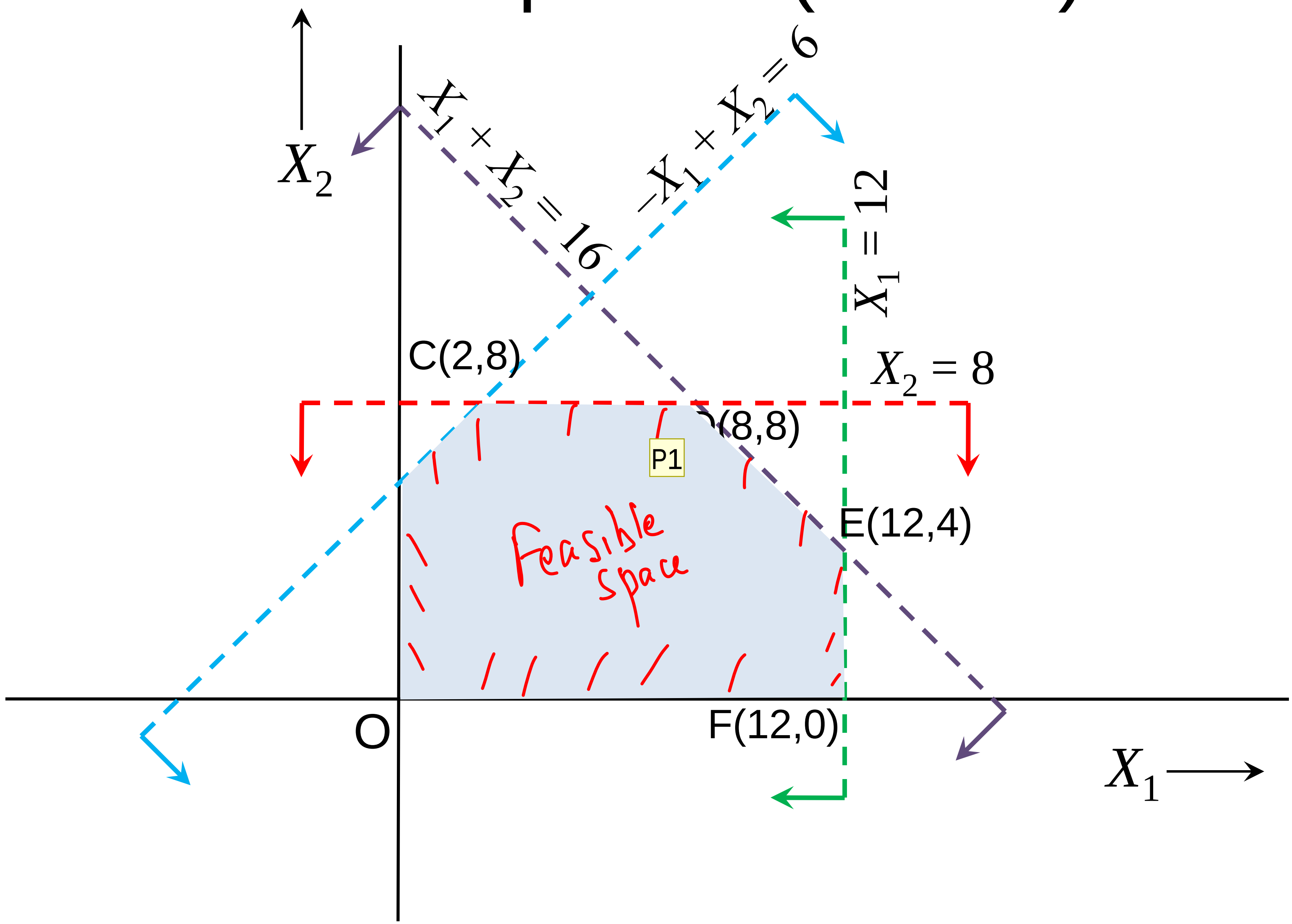
s.t.

$$-X_1 + X_2 \leq 6$$
$$X_1 \leq 12$$
$$X_1 + X_2 \leq 16$$
$$X_2 \leq 8$$
$$X_1, X_2 \geq 0$$

Max $Z = w_1 Z_1 + w_2 Z_2$

s.t

Example – 1 (Contd.)



Example – 1 (Contd.)

Weighting method:

- Weights are assigned to the OFs

$$Z = w_1 Z_1 + w_2 Z_2$$

$$Z = w_1 \left\{5X_1 - 4X_2\right\} + w_2 \left\{-2X_1 + 8X_2\right\}$$

- The solution for different weights are examined.

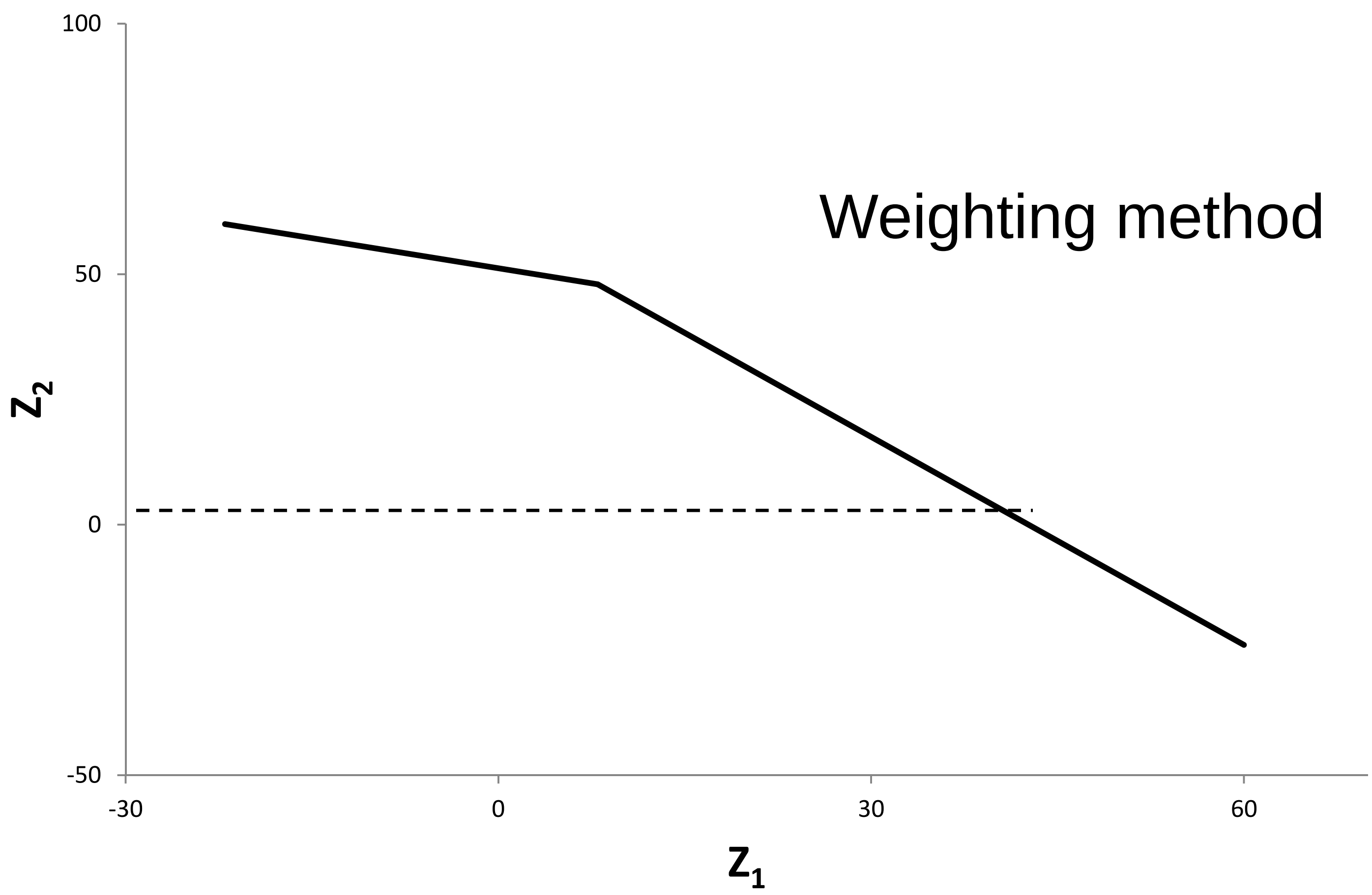
Example – 1 (Contd.)

$$Z = w_1\{5X_1 - 4X_2\} + w_2\{-2X_1 + 8X_2\}$$

w_1	w_2	Z	Max value of Z	X_1	X_2	Z_1	Z_2
1	1	$X_1 + 12X_2$	56	8	8	8	48
1	2	$X_1 + 12X_2$	104	8	8	8	48
1	3	$-X_1 + 20X_2$	158	2	8	-22	60
1	4	$-3X_1 + 28X_2$	218	2	8	-22	60
1	5	$-5X_1 + 36X_2$	278	2	8	-22	60
2	1	$8X_1$	96	12	0	60	-24
3	1	$13X_1 - 4X_2$	156	12	0	60	-24
4	1	$18X_1 - 8X_2$	216	12	0	60	-24
5	1	$23X_1 - 12X_2$	276	12	0	60	-24

Example – 1 (Contd.)

Non-inferior solutions (Efficiency frontier)



Example – 1 (Contd.)

Constraint method:

- The problem is modified as

$$Z_1(X) = 5X_1 - 4X_2$$

$$Z_2(X) = -2X_1 + 8X_2$$

Max $\quad Z_1(X) = 5X_1 - 4X_2$

s.t.

$$-2X_1 + 8X_2 \geq L_2$$

$$-X_1 + X_2 \leq 6$$

$$X_1 \leq 12$$

$$X_1 + X_2 \leq 16$$

$$X_2 \leq 8$$

$$X_1, X_2 \geq 0$$

Constraint on Z_2

is Minimum level to which Z_2 must be satisfied.

Example – 1 (Contd.)

- Any optimal solution for an assumed value of L_2 is a noninferior solution, if the constraints with L_2 on the right-hand side is binding.
- By varying the value of L_2, we get different noninferior solutions.

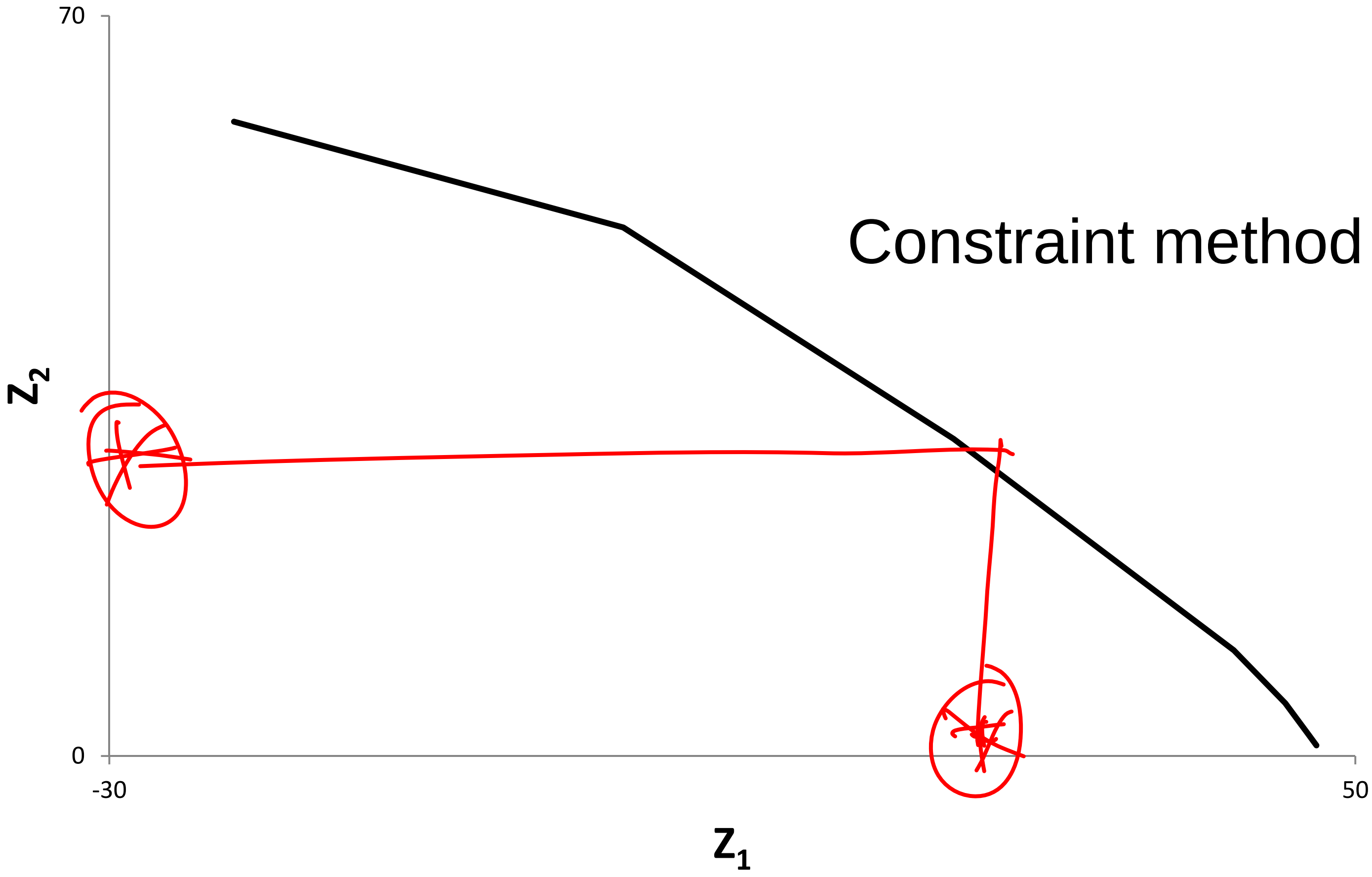
Example – 1 (Contd.)

L_2	Z_1	X_1	X_2
1	47.5	12	3.125
2	47	12	3.25
5	45.5	12	3.625
10	42.2	11.8	4.2
20	33.2	10.8	5.2
30	24.2	9.8	6.2
50	3	7	8
55	-9.5	4.5	8
60	-22	2	8

The constraint containing L_2 is binding in all the cases.

Example – 1 (Contd.)

Non-inferior solutions (Efficiency frontier)



Example – 1 (Contd.)

- The problem is solved with second constraint as OF

$$Z_1(X) = 5X_1 - 4X_2$$

$$Z_2(X) = -2X_1 + 8X_2$$

Max $Z_2(X) = -2X_1 + 8X_2$

s.t.

$$5X_1 - 4X_2 \geq L_1$$

$$-X_1 + X_2 \leq 6$$

$$X_1 \leq 12$$

$$X_1 + X_2 \leq 16$$

$$X_2 \leq 8$$

$$X_1, X_2 \geq 0$$

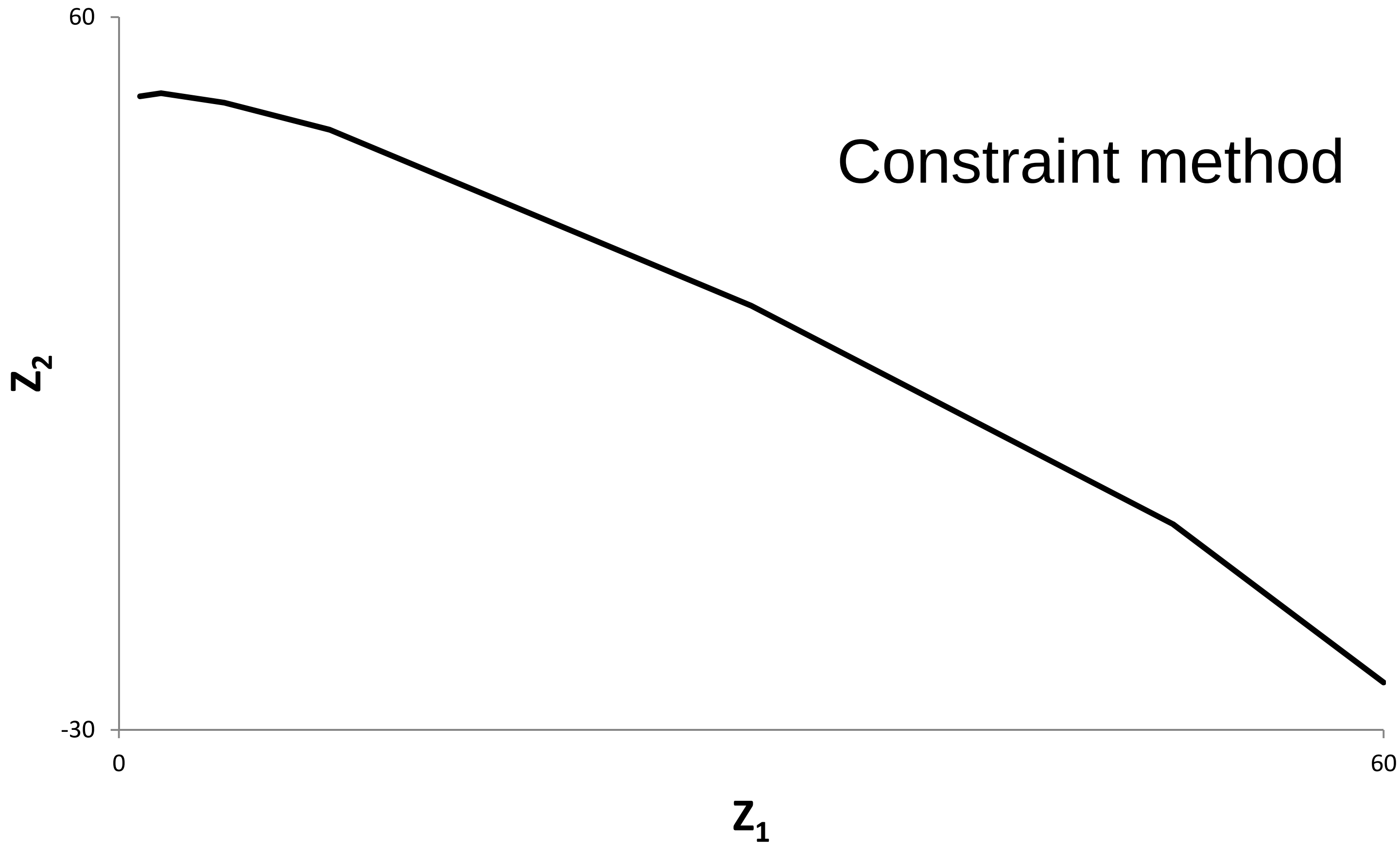
Example – 1 (Contd.)

L_1	Z_2	X_1	X_2
1	50	6.6	8
2	50.4	6.8	8
5	49.2	7.4	8
10	45.78	8.22	7.78
20	34.67	9.33	6.67
30	23.56	10.44	5.56
50	-4	12	2.5
55	-14	12	1.25
60	-24	12	0

The constraint containing L_1 is binding in all the cases.

Example – 1 (Contd.)

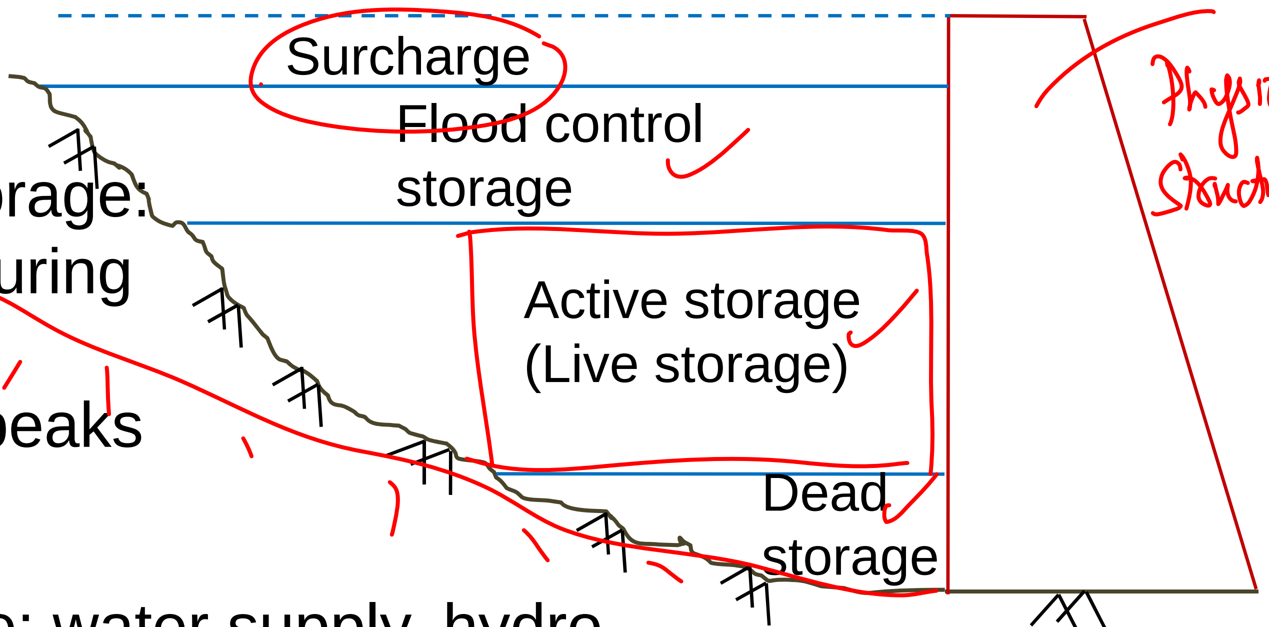
Non-inferior solutions (Efficiency frontier)



RESERVOIR SYSTEMS – DETERMINISTIC INFLOW

Reservoir System

- The total storage divided into three components:



Flood control storage: to retain water during flood events, to attenuate flood peaks d/s

Active storage: water supply, hydro power, irrigation, navigation etc.

Dead storage: sediment collection and recreation.

Reservoir Systems – Deterministic Inflows

Reservoir systems

- Reservoir modeling with deterministic inputs.
- Model formulations for two important aspects:
    - Reservoir sizing ✓
    - Reservoir operation ✓