INDIAN INSTITUTE OF SCIENCE

Water Resources Systems:
Modeling Techniques and Analysis

Lecture - 21

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Summary of the previous lecture

- Reservoir Sizing
  - Sequent peak analysis neglecting evaporation

$$K_t = K_{t-1} + R_t - Q_t \quad \text{......... if positive}$$
$$= 0 \quad \text{......... otherwise}$$

K = max{K_t}

  - Reservoir capacity using LP

Losses

Storage continuity

Min K_a

s.t. $S_t + Q_t - R_t - L_t = S_{t+1} \quad \forall t$

$$S_t \leq K_a \quad \forall t$$

$$S_t \geq 0; \quad K_a \geq 0$$

Reservoir Capacity Using LP

t

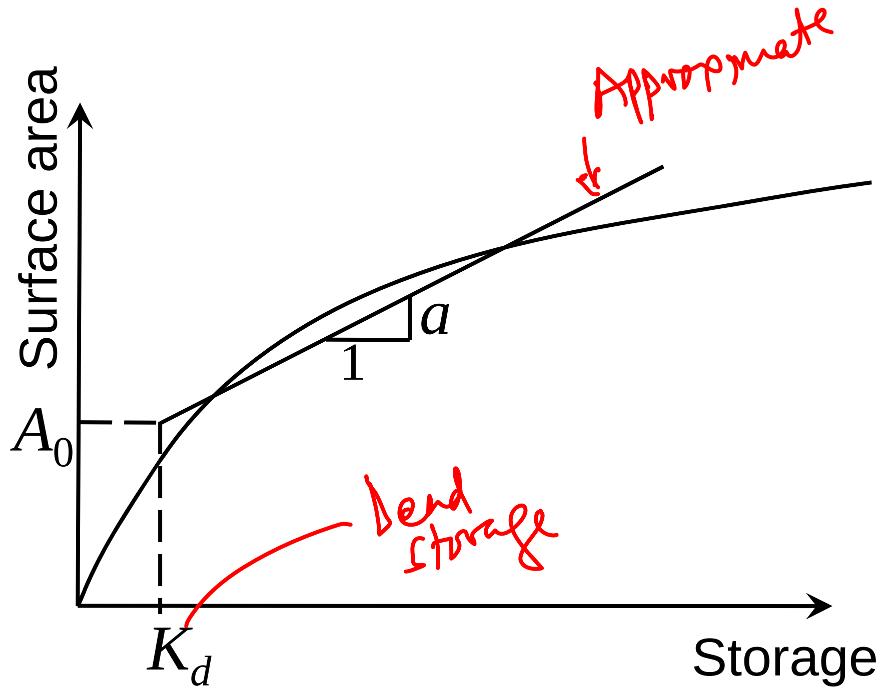
$S_t \quad S_{t+1}$

Continuity, with evaporation loss accounted

K_d : dead storage

A_0 : Surface area at dead storage

a : area per unit active storage above A_0.



Total evaporation in period t is given by

$$E_t = A_0 e_t + a\left(\underbrace{\frac{S_t + S_{t+1}}{2}}\right) e_t$$

Average storage in period t.

Reservoir Capacity Using LP

$$E_t = A_0 e_t + a_t \left(S_t + S_{t+1} \right) \qquad a_t = a\, e_t / 2$$

$$E_t = L_t + a_t \left(S_t + S_{t+1} \right)$$

where

L_t is the fixed evaporation loss = $e_t A_0$

e_t is the evaporation rate in period t

A_0 is water surface area at top of the dead storage level

a is the surface area per unit active storage (slope of the area-capacity relationship beyond the dead storage level).

Reservoir Capacity Using LP

Continuity, with evaporation loss accounted

$$S_t + Q_t - R_t - E_t = S_{t+1} \qquad \forall t$$

Substitute in E_t continuity equation

$$E_t = L_t + a_t\,(S_t + S_{t+1})$$

$$L_t = e_t A_0$$

$$S_t + Q_t - R_t - E_t = S_{t+1}$$

$$S_t + Q_t - R_t - \left\{L_t + a_t\left(S_t + S_{t+1}\right)\right\} = S_{t+1}$$

$$S_t + Q_t - R_t - A_0 e_t - a_t S_t - a_t S_{t+1} = S_{t+1}$$

$$\left(1 - a_t\right)S_t + Q_t - R_t - A_0 e_t = \left(1 + a_t\right)S_{t+1}$$

$$\boxed{\left(1 + a_t\right)S_{t+1} - \left(1 - a_t\right)S_t = Q_t - R_t - A_0 e_t}$$

Continuity equation, with storage dependent losses accounted for

Reservoir Capacity Using LP

Constraints: The storage constraint is

$$(1 - a_t)\, S_t + Q_t - L_t - R_t = (1 + a_t)\, S_{t+1} \quad \forall t$$

The storage in each period is bound by the capacity, K, thus

$$S_t \leq K \qquad \forall t$$

Also $R_t \geq D_t \qquad \forall t$

Specify D_t'

Release should be at least equal to D_t

The reservoir capacity K is a variable, which must be minimized. The objective is to minimize K.

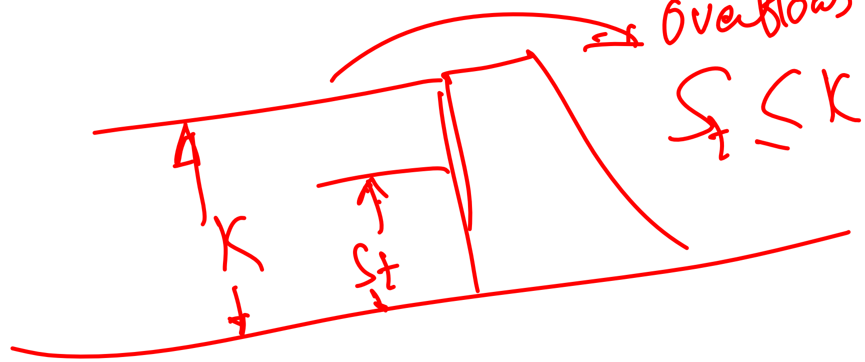
Reservoir Capacity Using LP

The model formulation reduces to

Live Storage

Overflows

$S_t \le K$



Minimize K

s.t.

$$(1 - a_t)\, S_t + Q_t - L_t - R_t = (1 + a_t)\, S_{t+1} \qquad \forall t$$

$$S_t \le K \qquad \forall t$$

$$R_t \ge D_t \qquad \forall t$$

$S_{T+1} = S_1$, where T is the last period in sequence

Reservoir Capacity Using LP

- The last constraint means that when considering a sequence of monthly inflows in a year, $T = 12$, S_{13} in the formulation is set equal to S_1.

- The purpose is to ensure that the storage at the end of the last period in the year is same as the storage at the beginning of the first period as the inflow sequence is assumed to be repetitive.

- In the storage continuity equation, spill, if any in period t, is absorbed in the term R_t.

Example – 1

The monthly inflows (Q_t) and demands (D_t) in Mm^3 and evaporation rate (e_t) in mm for a reservoir are given below

	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Q_t	70.61	412.75	348.40	142.29	103.78	45.00	19.06
D_t	51.68	127.85	127.85	65.27	27.18	203.99	203.99
e_t	231.81	147.57	147.57	152.14	122.96	121.76	99.89

	Jan.	Feb.	Mar.	Apr.	May
Q_t	14.27	10.77	8.69	9.48	18.19
D_t	179.47	89.76	0	0	0
e_t	97.44	106.14	146.29	220.97	246.75

Area corresponding to dead storage level, A_0 = 37.01 Mm^2

Slope of the area-capacity curve beyond dead storage, a = 0.117115 m^2/m^3

Example – 1 (Contd.)

e_t converted to ‘m’

given

Month	Q_t (Mm^3)	D_t (Mm^3)	e_t mm	$a_t=a*e_t/2$	$L_t=A_o*e_t$ (Mm^3)	$(1 - a_t)$	$(1 + a_t)$
Jun	70.61	51.68	231.81	0.01357	8.58	0.9864	1.0136
Jul	412.75	127.85	147.57	0.00864	5.46	0.9914	1.0086
Aug	348.4	127.85	147.57	0.00864	5.46	0.9914	1.0086
Sep	142.29	65.27	152.14	0.00891	5.63	0.9911	1.0089
Oct	103.78	27.18	122.96	0.00720	4.55	0.9928	1.0072
Nov	45	203.99	121.76	0.00713	4.51	0.9929	1.0071
Dec	19.06	203.99	99.89	0.00585	3.70	0.9942	1.0058
Jan	14.27	179.47	97.44	0.00571	3.61	0.9943	1.0057
Feb	10.77	89.76	106.14	0.00622	3.93	0.9938	1.0062
Mar	8.69	0	146.29	0.00857	5.41	0.9914	1.0086
Apr	9.48	0	220.97	0.01294	8.18	0.9871	1.0129
May	18.19	0	246.75	0.01445	9.13	0.9856	1.0144

Example – 1 (Contd.)

Minimize K

s.t. $(1 - a_t)\, S_t + Q_t - L_t - R_t = (1 + a_t)\, S_{t+1}$ $\quad R_t \geq D_t \quad S_t \leq K$

$0.9864*S_1 + 70.61 - 8.58 - R_1 = 1.0136*S_2 \quad R_1 \geq 51.68 \quad S_1 \leq K$

$0.9914*S_2 + 412.75 - 5.46 - R_2 = 1.0086*S_3 \quad R_2 \geq 127.85 \quad S_2 \leq K$

$0.9914*S_3 + 348.4 - 5.46 - R_3 = 1.0086*S_4 \quad R_3 \geq 127.85 \quad S_3 \leq K$

$0.9911*S_4 + 142.29 - 5.63 - R_4 = 1.0089*S_5 \quad R_4 \geq 65.27 \quad S_4 \leq K$

$0.9928*S_5 + 103.78 - 4.55 - R_5 = 1.0072*S_6 \quad R_5 \geq 27.18 \quad S_5 \leq K$ $\quad S_{13} = S_1$

$0.9929*S_6 + 45 - 4.51 - R_6 = 1.0071*S_7 \quad R_6 \geq 203.99 \quad S_6 \leq K$

$0.9942*S_7 + 19.06 - 3.7 - R_7 = 1.0058*S_8 \quad R_7 \geq 203.99 \quad S_7 \leq K$

$0.9943*S_8 + 14.27 - 3.61 - R_8 = 1.0057*S_9 \quad R_8 \geq 179.47 \quad S_8 \leq K$

$0.9938*S_9 + 10.77 - 3.93 - R_9 = 1.0062*S_{10} \quad R_9 \geq 89.76 \quad S_9 \leq K$

$0.9914*S_{10} + 8.69 - 5.41 - R_{10} = 1.0086*S_{11} \quad R_{10} \geq 0 \quad S_{10} \leq K$

$0.9871*S_{11} + 9.48 - 8.18 - R_{11} = 1.0129*S_{12} \quad R_{11} \geq 0 \quad S_{11} \leq K$

$0.9856*S_{12} + 18.19 - 9.13 - R_{12} = 1.0144*S_1 \quad R_{12} \geq 0 \quad S_{12} \leq K$

Example – 1 (Contd.)

Solution:

K = 617.928 Mm^3

Required Capacity

Month	S_t (Mm^3)	R_t (Mm^3)
Jun	13.26	68.13
Jul	6.9	127.85
Aug	283.83	127.85
Sep	492.24	65.27
Oct	554.32	27.2
Nov	617.93	203.99
Dec	446.87	203.99
Jan	254.2	179.5
Feb	83.44	89.76
Mar	0	0
Apr	3.252	0
May	4.45	0

Example – 1 (Contd.)

<u>Without evaporation</u>:

K = 588.11 Mm^3

$S_{t+1} = S_t + Q_t - R_t \quad \forall t$

Min K

$S_t \le K$

Month	S_t (Mm^3)	R_t (Mm^3)
Jun	0	70.61
Jul	0	198.81
Aug	213.94	127.85
Sep	434.49	65.27
Oct	511.51	27.18
Nov	588.11	203.99
Dec	429.12	203.99
Jan	244.19	179.47
Feb	78.99	89.76
Mar	0	8.69
Apr	0	9.48
May	0	18.19

Reservoir Capacity Using LP

Tips in model formulation using LP

1. Equality form of constraints:

   $$(1 - a_t)\, S_t + Q_t - L_t - R_t = (1 + a_t)\, S_{t+1} \qquad \forall t$$

- The constraint should imply that R_t includes the demand to be met and possible spill in the period.

$$R_t \geq D_t$$

Ensure demand in every period is satisfied in full

Reservoir Capacity Using LP

2. Inequality constraints:

- Use ' $\geq$ ' instead of ' = ' and substitute D_t in place of R_t

$$(1 - a_t)\, S_t + Q_t - L_t - D_t \geq (1 + a_t)\, S_{t+1} \qquad \forall t$$

- Builds in flexibility to take care of spills.
- Excess water over demand and evaporation is stored within the reservoir until the capacity.

Reservoir Capacity Using LP

The problem statement is

Minimize K

s.t.

$$(1 - a_t)\, S_t + Q_t - L_t - D_t \geq (1 + a_t)\, S_{t+1} \qquad \forall t$$

$$S_t \leq K \qquad \forall \quad t$$

The spill ($Spill_t$) in period t is

$$Spill_t = (1 - a_t)\, S_t + Q_t - L_t - D_t - a_t S_{t+1} - K \qquad \text{…if positive}$$

$$= 0 \qquad \text{… otherwise}$$

$$\forall t$$

Reservoir Capacity Using LP

3. Other equality form of constraints:

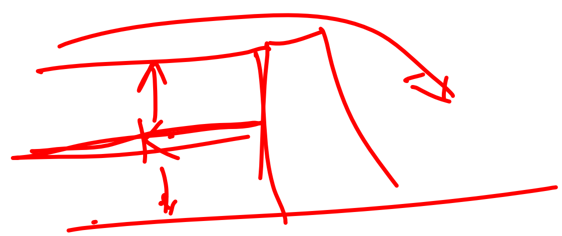
- Specify additional term for spill in each constraint and penalize spills in the OF

Additional variable

$$(1 - a_t)\, S_t + Q_t - L_t - D_t - Spill_t = (1 + a_t)\, S_{t+1} \qquad \forall\, t$$

$S_t \leq K$

The modified OF is



Minimize $\quad K + M \sum_t Spill_t$

Large value.

Reservoir Capacity Using LP

Storage yield function:

- To determine the maximum constant yield (constant release in all periods within a year) from a reservoir.
- Formulation is

K is given and R is to be determined.

$\boxed{R_1 = R_2 = \cdots = R_T = R}$

Constant demand

Maximize R

s.t.

Decision Variables

Decision Variable

$$(1 - a_t)\, S_t + Q_t - L_t - R \geq (1 + a_t)\, S_{t+1} \quad \forall t$$

and

$$S_t \leq K \qquad \forall t$$

Known.

with $S_{T+1} = S_1$, where T is the last period

Example – 2

Solve the problem in Example-1 with constant storage capacity of ~~600~~ 617... Mm^3

LP Formulation is

Maximize R

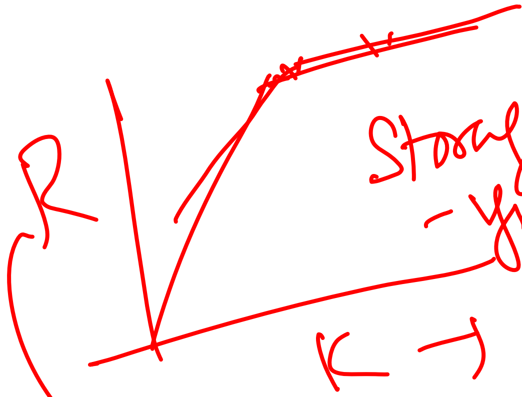
s.t.

$$(1 - a_t)\, S_t + Q_t - L_t - R \geq (1 + a_t)\, S_{t+1}$$

and

$$S_t \leq K \qquad t = 1, 2, \ldots 12$$

$$S_{13} = S_1$$



Storage - yield function

$\Rightarrow$ Yield of the Reservoir