INDIAN INSTITUTE OF SCIENCE

Water Resources Systems:
Modeling Techniques and Analysis

Lecture - 26

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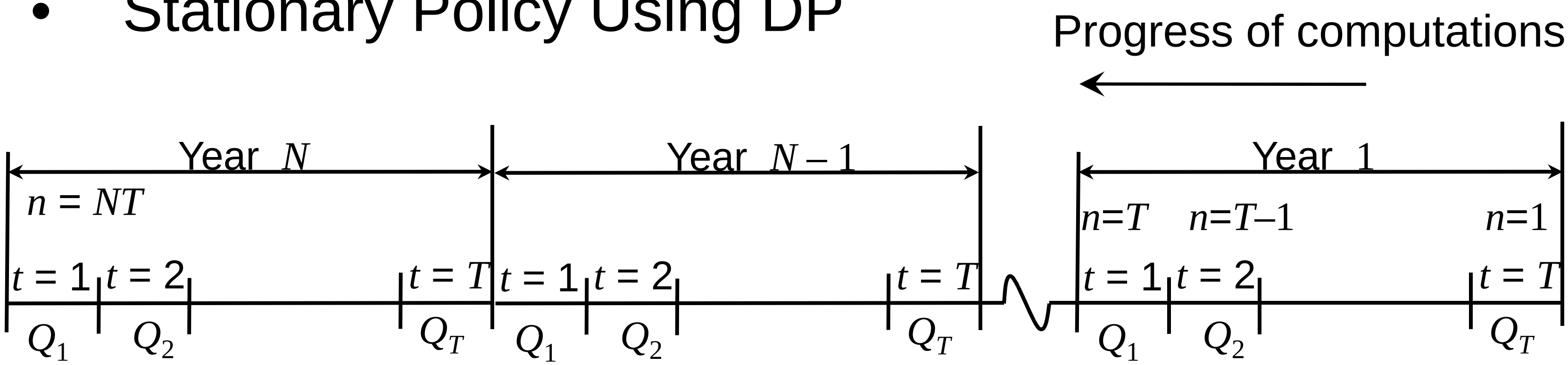
Summary of the previous lecture

- Stationary Policy Using DP



$$f_t^n(S_t) = \max\left[B_t(R_t, S_t) + f_{t+1}^{n-1}(S_t + Q_t - R_t)\right]$$

$$0 \le R_t \le S_t + Q_t \quad ; \quad S_t + Q_t - R_t \le K$$

Steady state:

$$\left[f_t^{n+T}(S_t) - f_t^n(S_t)\right]$$

remains constant $\forall S_t$

- Hydropower Generation

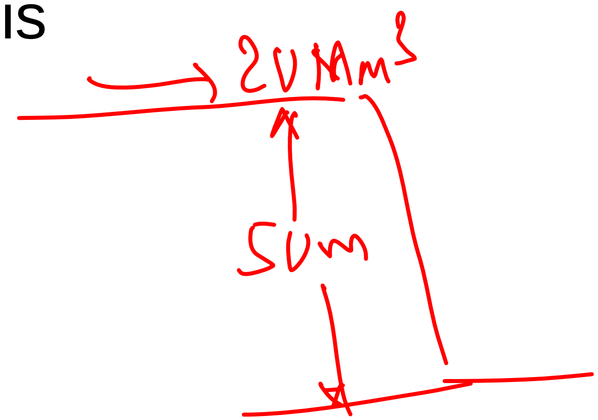
$$kWH_t = 2725 \; R_t \; H_t \; \eta$$

  - Firm power ; Secondary power ; Run-of-the-river power plants

Hydropower Generation

- For example, a river with a minimum monthly flow of 20 Mm^3 has a drop of 50 m at a site along the river. Consider the overall efficiency for power generation as 75%

Firm energy produced at that site is



$$kWH_t = 2725\ R_t\ H_t\ \eta$$

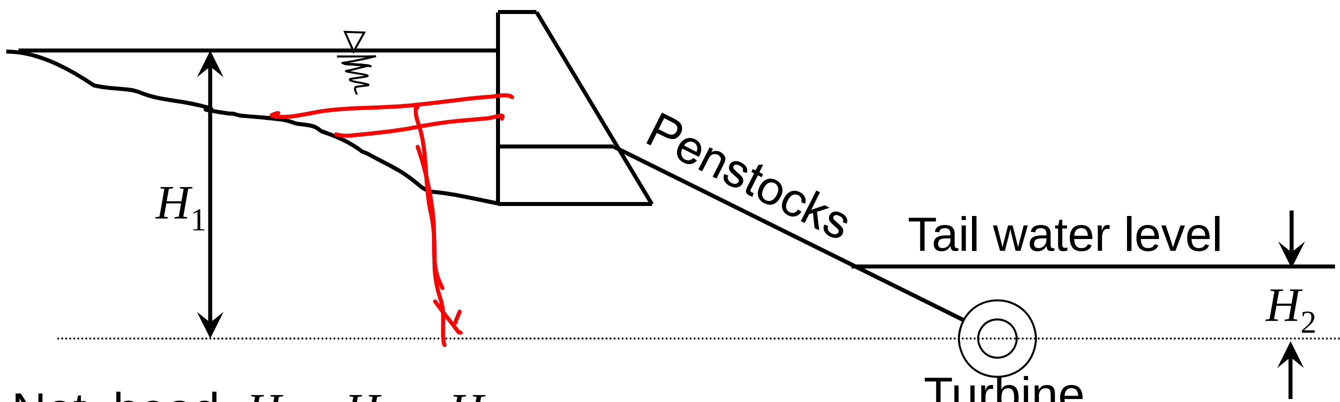
$$= 2725 \times 20 \times 50 \times 0.75$$

$$= 2043750\ \ kWH$$

$$= 2.044\quad GWH$$ (giga watt hour)

Hydropower Generation

Simulation of reservoir operation for hydropower generation:



Net head, $H_t = H_1 - H_2$

- The data required is
  - The inflow series at the reservoir
  - Storage-elevation-area relationships
  - Power plant efficiency.

Hydropower Generation

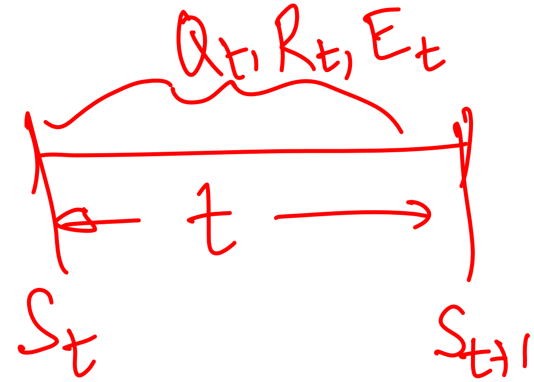
Simulation for monthly operation:

$$kWH_t = 2725\ R_t\ H_t\ \eta$$

Rewrite the expression for a period of one month

$$P = \frac{2725\ R_t\ H_t\ \eta}{1000 \times 30 \times 24}$$

$$= 0.003785\ R_t\ H_t\ \eta \quad MW$$



$$R_t = \frac{P}{0.003785\ H_t\ \eta}$$

R_t is release to penstock in Mm^3 in period t, P is power in MW, H_t is the net head in m in period t and η is plant efficiency.

Hydropower Generation

Reservoir storage continuity:

Area of water spread

$$S_{t+1} = S_t + Q_t - E_t - R_t - Spill_t$$

where

S_t is the storage at the beginning of the period t

Q_t is the reservoir inflow during the period t

R_t is the release required in the period t to generate the specified power corresponding to the net head, resulting from the average of S_t and S_{t+1}.

E_t is the evaporation loss in the period t corresponding to water spread area at the average storage.

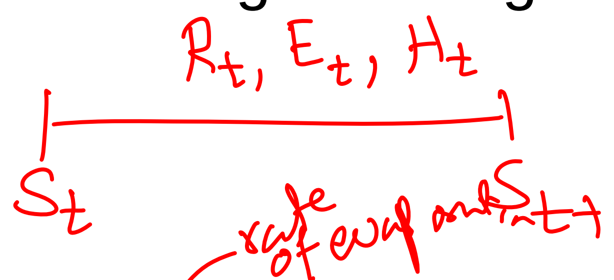
$Spill_t$ is the spill (overflow) during the period t

Hydropower Generation

Procedure for obtaining H_t, R_t, E_t and S_{t+1} iteratively:

1. Assume average storage $\bar{S}_t = S_t$ (Known)
2. Corresponding to $\bar{S}_t$, obtain net head, H_t, and water spread area, A_t, from storage-elevation-area relationships.
3. Determine the release, R_t, required for generating the specified power, P, from

$$R_t = \frac{P}{0.003785\ H_t\ \eta}$$



4. Obtain the evaporation loss from $E_t = e_t A_t$ where e_t is the evaporation rate in period t and A_t corresponds to the storage $\bar{S}_t$.

Hydropower Generation

5. Get the end of period storage,

$$S_{t+1} = S_t + Q_t - E_t - R_t \quad \text{if } S_{t+1} < \text{reservoir capacity, } K.$$

$$= K \text{ ,} \quad \text{otherwise}$$

6. Get the average storage, $\bar{S}_t^* = \dfrac{S_t + S_{t+1}}{2}$

7. If $\bar{S}_t^*$ is nearly equal to $\bar{S}_t$, the computed values of H_t, R_t, E_t and S_{t+1} are acceptable.

   Else, set $\bar{S}_t = \bar{S}_t^*$ and go to step 2; repeat steps 2 to 7 until the computed values of H_t, R_t, E_t and S_{t+1} are acceptable.

Example – 2

Simulate the reservoir operation for hydropower generation with the following data:

The monthly inflows (Q_t) in Mm^3 and evaporation rate (e_t) in cm for the reservoir are as follows

	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Q_t	190.76	433.76	212.97	146.89	209.72	42.92	28.02
e_t	11	9	8	9	8	7	8

	Jan.	Feb.	Mar.	Apr.	May
Q_t	11.95	7.07	9.25	9.89	65.16
e_t	8	10	13	14	11

Example – 2 (Contd.)

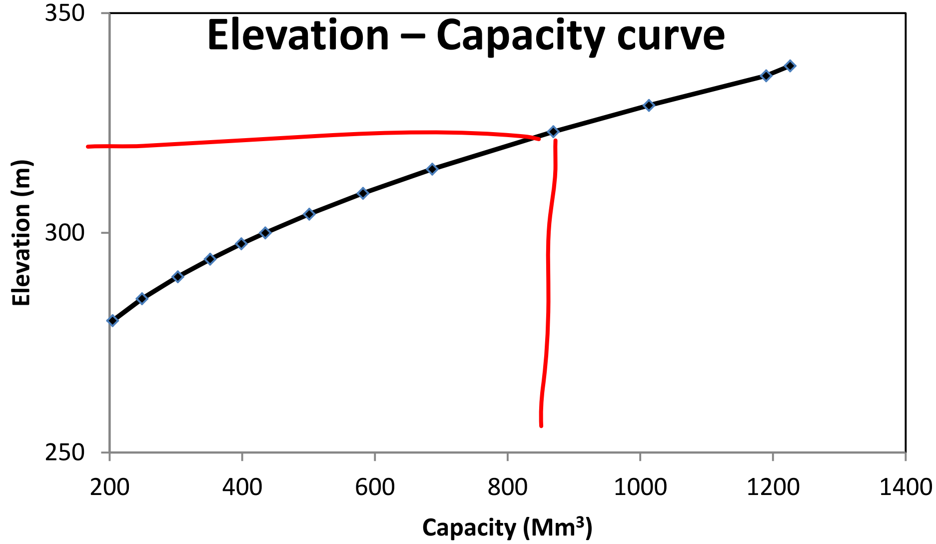
Reservoir capacity,
K = 1226 Mm^3

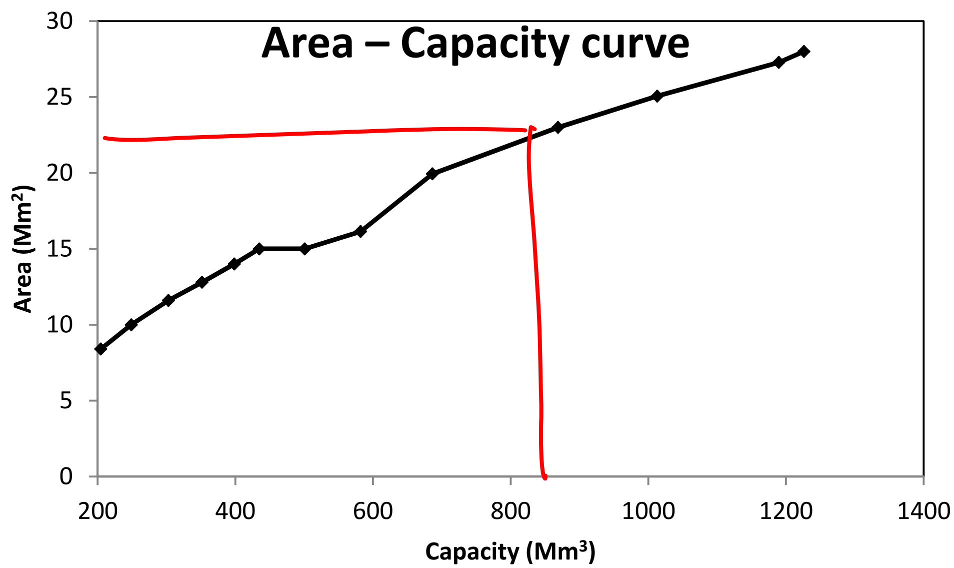
Minimum power desired in a month,
P = 73.5 MW

Plant efficiency = 81.54%
Initial storage = 824.63 Mm^3
Tailrace water level = 47 m

Capacity (Mm^3)	Elevation (m)	Area (Mm^2)
204.5	280	8.4
248.82	285	10
302.82	290	11.6
351.62	294	12.8
398.52	297.5	14
434.77	300	15
500.94	304.25	15
582.02	309	16.14
686.36	314.5	19.94
868.85	323	23
1013.03	329	25.06
1189.68	335.75	27.28
1226	338	28

Example – 2 (Contd.)





Example – 2 (Contd.)

Illustration of calculations for June month (t = 1):

- Initial storage, S_1 = 824.63 Mm^3; Assume $\bar{S}_t = 824.63 \ Mm^3$
- Inflow for June month = 190.76 Mm^3
- From the Storage – elevation – area data corresponding to the initial storage, S_1=824.63 Mm^3, total head = 320.84 m and A_t = 22.26 Mm^2
- Net head, H_t = 320.84 – 47 = 273.94 m
- For P = 73.5 MW and η = 81.54%,

$$R_t = \frac{P}{0.003785 \ H_t \ \eta}$$

$$= \frac{73.5}{0.003785 \times 273.94 \times 0.8154} = 86.935 \, Mm^3$$

Example – 2 (Contd.)

- Evaporation rate, e_t for June month = 11 cm = 0.11m
- Evaporation loss corresponding to e_t = 0.11 m and A_t = 22.26 Mm^2 is

$$E_t = e_t A_t = 0.11 \times 22.26 = 2.003 \text{ Mm}^3$$

- End of period storage, S_{t+1} is calculated as

$$S_{t+1} = S_t + Q_t - E_t - R_t$$

$$= 824.63 + 190.76 - 2.003 - 86.935$$

$$= 926.45 \; Mm^3 \qquad S_{t+1} < K \; (1226 \text{ Mm}^3)$$

Average storage, $\bar{S}_t^* = \dfrac{S_t + S_{t+1}}{2} = \dfrac{824.63 + 926.45}{2} = 875.5$

$$\bar{S}_t^* \neq \bar{S}_t$$

Example – 2 (Contd.)

$\bar{S}_t = 875.5$

Second iteration for June month:

- Corresponding to $\bar{S}_t^* = 875.5$ Mm^3, total head = 323.3 m and $A_t = 23.1$ Mm^2
- Net head, $H_t = 323.3 - 47 = 276.3$
- For $P = 73.5$ MW and $\eta = 81.54\%$,

$$R_t = \frac{73.5}{0.003785 \times 276.3 \times 0.8154} = 86.2\, Mm^3$$

- Evaporation loss is

$E_t = e_t A_t = 0.11 \times 23.1 = 2.08$ Mm^3

- End of period storage, S_{t+1} is

$$S_{t+1} = 824.63 + 190.76 - 2.08 - 86.2 = 927.11\ Mm^3$$

$S_{t+1} < K$ (1226 Mm^3)

Example – 2 (Contd.)

Average storage, $\bar{S}_t^* = \dfrac{875.5 + 927.11}{2} = 901.3$ Mm³

$$\bar{S}_t^* \neq \bar{S}_t$$

Set $\bar{S}_t = 901.3$

Other iterations are performed in the same line for June month until

$$\bar{S}_t^* = \bar{S}_t$$

$t = 1$

Final solution for June month is

R_t = 85.53 Mm^3
H_t = 278.45 m
E_t = 2.15 Mm^3 and
S_{t+1} = 927.72 Mm^3

Convergence

Example – 2 (Contd.)

End-of-storage for June month is initial storage for July month.

S_2 = 927.72 Mm^3

Same procedure is followed for obtaining the H_t, R_t, E_t and S_{t+1} values for July month.

Final solution for July month (t = 2) is

R_t = 81.84 Mm^3
H_t = 291 m
E_t = 2.52 Mm^3 and
S_{t+1} = 1226 Mm^3

Example – 2 (Contd.)

Iterative Procedure

Simulation results

Month	Q_t Mm³	e_t cm	S_t Mm³	H_t m	A_t Mm²	R_t Mm³	E_t Mm³	$Spill_t$ Mm³	S_{t+1} Mm³
Jun	190.76	11	824.63	278.45	23.84	85.53	2.15	0	927.72
Jul	433.76	9	927.72	291.00	28.00	81.84	2.52	47.32	1226.00
Aug	212.97	8	1226.00	291.00	28.00	81.84	2.24	128.89	1226.00
Sep	146.89	9	1226.00	291.00	28.00	81.84	2.52	62.53	1226.00
Oct	209.72	8	1226.00	291.00	28.00	81.84	2.24	125.64	1226.00
Nov	42.92	7	1226.00	288.55	27.21	82.53	1.91	0	1184.48
Dec	28.02	8	1184.48	286.36	26.50	83.16	2.12	0	1127.22
Jan	11.95	8	1127.22	283.53	25.56	83.99	2.05	0	1053.13
Feb	7.07	10	1053.13	280.33	24.49	84.95	2.45	0	972.80
Mar	9.25	13	972.80	277.01	23.35	85.97	3.03	0	893.04
Apr	9.89	14	893.04	273.39	22.06	87.11	3.09	0	812.73
May	65.16	11	812.73	272.24	21.64	87.48	2.38	0	788.03

Hydropower Generation

Primary and additional power:

- When the power draft is adequate to generate the specified power P, the primary power is equal to P itself.
- When the power draft is less than that required to generate the power P, the primary power P is

$$P = 0.0030864 \; R_t \; H_t$$

- The additional power is generated only when the reservoir spills.

Hydropower Generation

- The spill during a month is computed based on end-of-the-period storage as,

$$Spill_t = 0 \quad \text{if} \quad S_t + Q_t - E_t - R_t \leq K$$
$$= S_t + Q_t - E_t - R_t - K \quad \text{otherwise}$$

- When there is a spill during a period, the end-of-the-period storage, S_{t+1}, is set to K, after computing the spill.
- The additional power is computed based on the spill with net head corresponding to full reservoir level as

$$P' = 0.0030864 \; Spill_t \; H_{\max}$$

H_{max} is the net head corresponding to full reservoir level

Example – 2 (Contd.)

In the previous example, the spill occurs in the months of Jul., Aug., Sep. and Oct.

H_{max} = 291 m

Month	$S_t + Q_t - E_t - R_t$ Mm³	K Mm³	$Spill_t$ Mm³	Additional power MW
Jul	1276.64	1226	47.32	42.50
Aug	1354.89	1226	128.89	115.76
Sep	1288.53	1226	62.53	56.16
Oct	1351.64	1226	125.64	112.84

$$P' = 0.0030864\ Spill_t\ H_{\max}$$