INDIAN INSTITUTE OF SCIENCE

Water Resources Systems:
Modeling Techniques and Analysis

Lecture - 27

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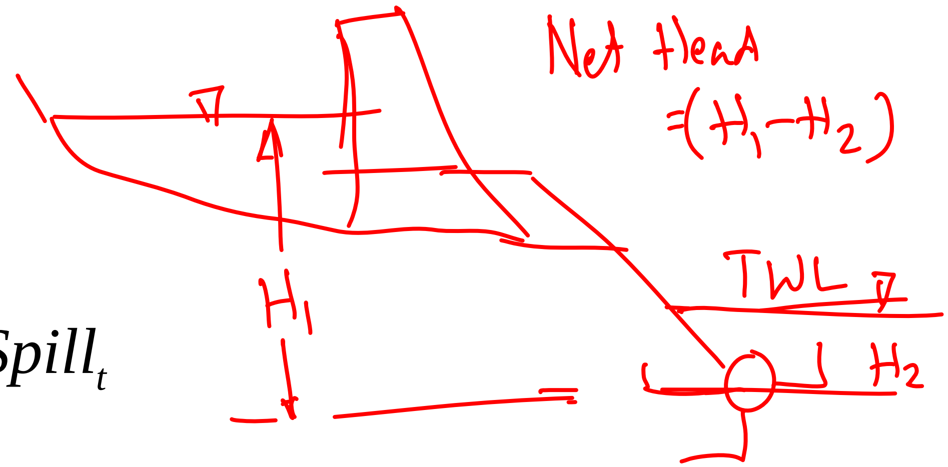
Summary of the previous lecture

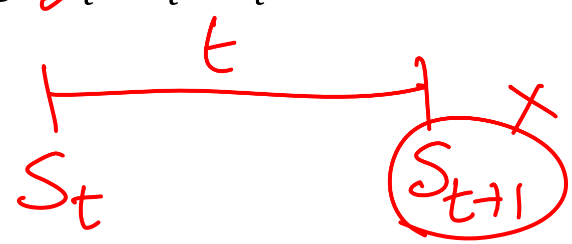
- Hydropower Generation
  - Simulation of reservoir operation for hydropower generation

$$R_t = \frac{P}{0.003785\ H_t\ \eta}$$



$$S_{t+1} = S_t + Q_t - E_t - R_t - Spill_t$$

  - Iterative procedure for obtaining H_t, R_t, E_t and S_{t+1}



  - Primary and additional power

RESERVOIR SYSTEMS – RANDOM INFLOWS

Reservoir Systems – Random Inflows

- Uncertainty in hydrologic variables (inflows, rainfall, evapotranspiration etc.)
- Two classical approaches to deal uncertainty in optimization models
  - Implicit Stochastic Optimization (ISO): optimization model deterministic; sequences of random inputs; large number of model runs.
  - Explicit Stochastic Optimization (ESO): optimization model stochastic; probability distributions of inputs; single run of model.
    - ➢ Chance Constrained Linear Programming (CCLP)
    - ➢ Stochastic Dynamic Programming (SDP)

Basic Probability Theory

NPTEL Course on 'Stochastic Hydrology'

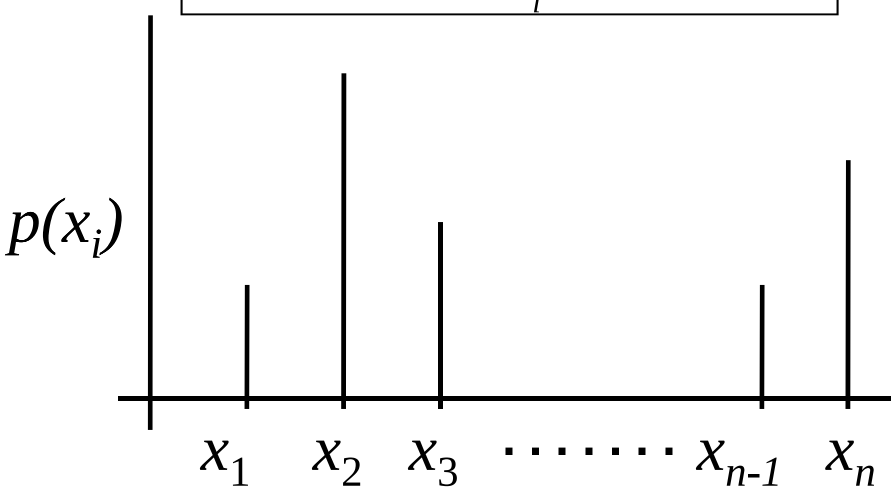
- Random variable: (intuitively) A RV is a variable whose value cannot be known with certainty, until the variable actually takes on a value.
- Discrete R.V.: Set of values a random variable can assume is finite (or countably infinite).

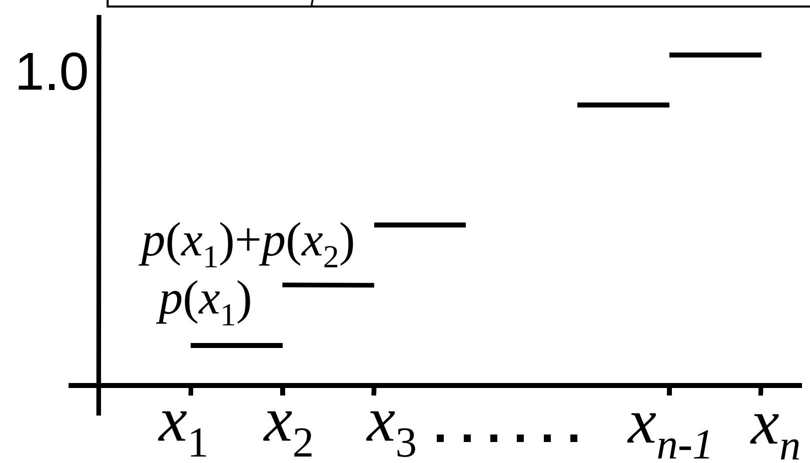
Probability Mass Function

p(x_i) $\geq$ 0 ; $\sum_{i} p(x_i) = 1$

Cumulative distribution function

$F(x) = \sum_{x_i \leq x} p(x_i)$





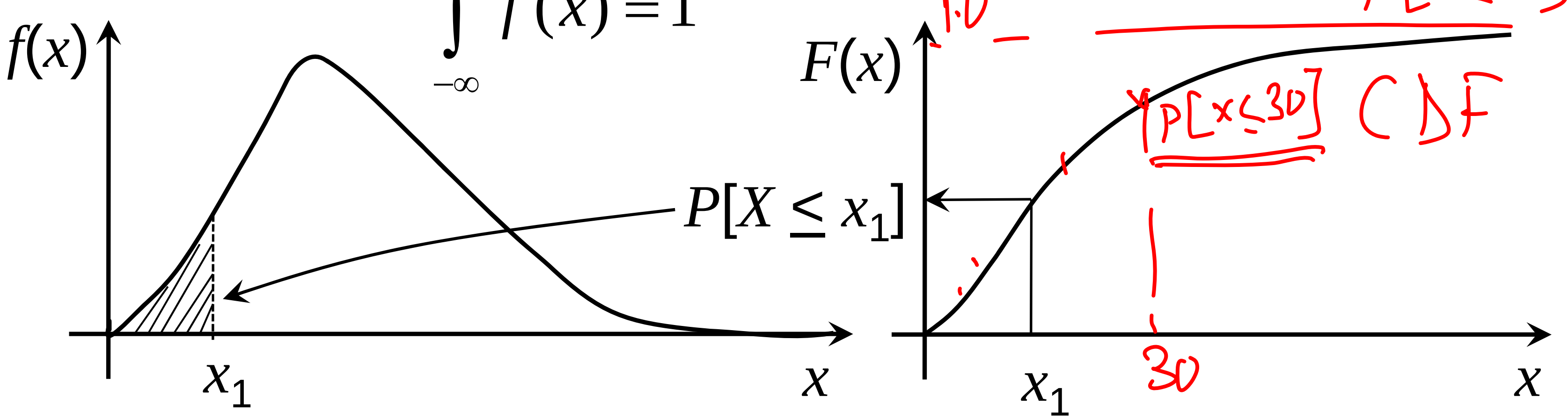
Basic Probability Theory

- Continuous R.V.: If the set of values a random variable can assume is infinite (the r.v. can take on values on a continuous scale)

$f(x) \geq 0$ and pdf

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$F(x) = \int_{-\infty}^{x} f(x)dx = P[X \leq x]$$



Probability Density Function

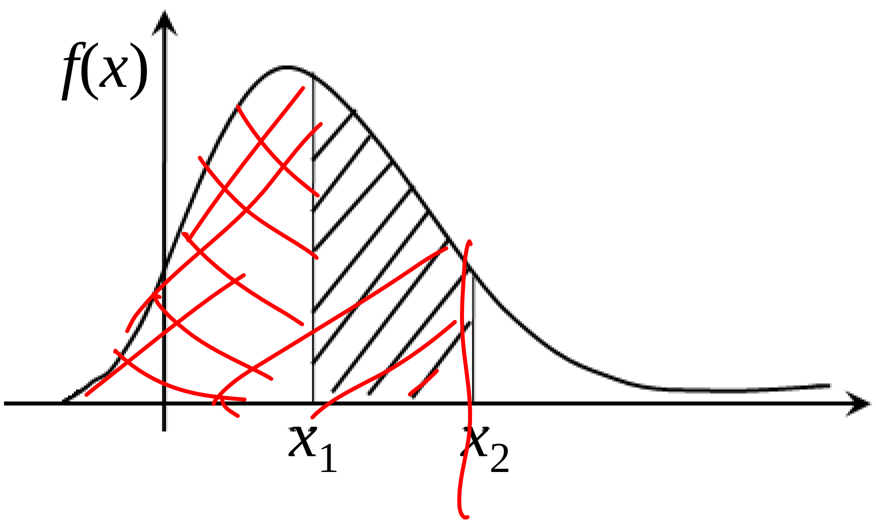
Cumulative Distribution Function

Basic Probability Theory

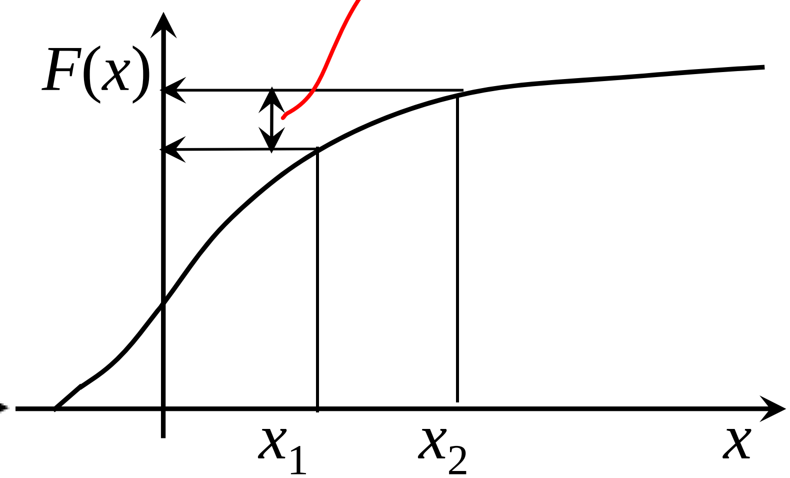
$$P[x_1 \le X \le x_2] = \int_{x_1}^{x_2} f(x)dx$$

$$P[x_1 \le X \le x_2] = F(x_2) - F(x_1)$$

$P[x_1 \le X \le x_2]$



PDF



CDF

Example – 1

Consider the following pdf

$f(x) \geq 0$

$$f(x) = 2e^{-2x} \qquad x \geq 0$$

$= 0$ elsewhere.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{0}^{\infty} 2e^{-2x}\,dx = 1$$

1. Derive the cdf
2. What is the probability that X lies between 1 and 2
3. Determine 'x' such that $P[X \leq x] = 0.5$
4. Determine 'x' such that $P[X \geq x] = 0.75$

Example – 1 (Contd.)

1. CDF:

$$F(x) = \int_{-\infty}^{x} f(x)dx = \int_{0}^{x} f(x)dx$$

$$= \int_{0}^{x} 2e^{-2x}dx$$

$$= \left[-e^{-2x}\right]_{0}^{x}$$

$$\boxed{F(x) = \left[1 - e^{-2x}\right]} \qquad x \geq 0$$

Example – 1 (Contd.)

2. $P[1 \leq X \leq 2]$ = $F(2) - F(1)$

$$F(2) = \left[1 - e^{-2\times 2}\right] = 0.982$$

$$F(1) = \left[1 - e^{-2\times 1}\right] = 0.865$$

$$P[1 \leq X \leq 2] = 0.982 - 0.865$$

$$= 0.117$$

$P[x_1 \leq X \leq x_2]$

$= F(x_2) - F(x_1)$

3. Determine 'x' such that $P[X \leq x] = 0.5$

$F(x)$

$$\boldsymbol{P[X \leq x]} = \left[1 - e^{-2x}\right] = 0.5$$

$$-2x = \ln 0.5$$

$$x = 0.35$$

Example – 1 (Contd.)

4. Determine ‘x’ such that $P[X \geq x] = 0.75$

$$P\left[X \geq x\right] = 1 - \underbrace{P\left[X \leq x\right]}_{F(x)} = 0.75$$

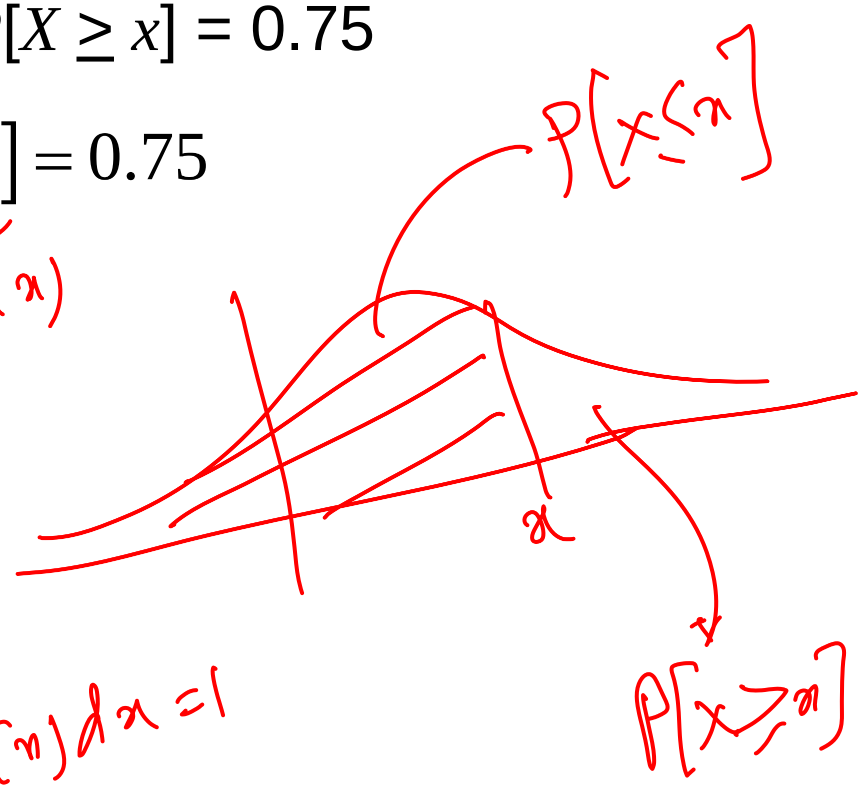
$$1 - \left[1 - e^{-2x}\right] = 0.75$$

$$e^{-2x} = 0.75$$

$$-2x = \ln 0.75$$

$$x = 0.144$$

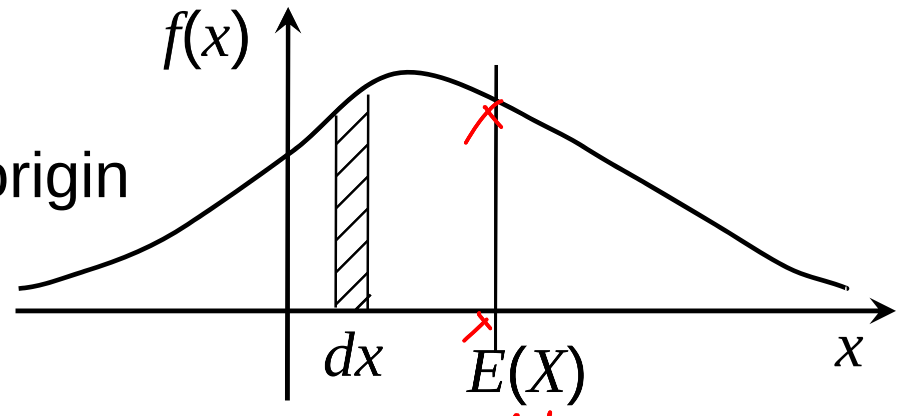
$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$



Basic Probability Theory

$E(X)$: Expected value of 'X'

: First moment about the origin



μ

$$\mu = E(X) = \int_{-\infty}^{\infty} x f(x) dx$$

Discrete case: $\mu = \sum_{i=1}^{n} x_i p(x_i)$ n: Sample size

$E[g(x)]$

Variance: Second moment about the mean

$= \int_{-\infty}^{\infty} g(x) f(x) dx$

$$\sigma^2 = E(X-\mu)^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx$$

Basic Probability Theory

Arithmatic mean

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Sample estimate - Variance :

$$s^2 = \frac{\sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2}{n-1}$$

n: No. of observations in the sample

Standard deviation:

$$\sigma = +\sqrt{\sigma^2}$$

$$s = +\sqrt{s^2}$$

+ve square root of variance

Coefficient of variation:

$$c_v = \frac{\sigma}{\mu}$$

$$= \frac{s}{\bar{x}}$$

sample estimate

NPTEL
'Stochastic Hydrology'