INDIAN INSTITUTE OF SCIENCE

Water Resources Systems:

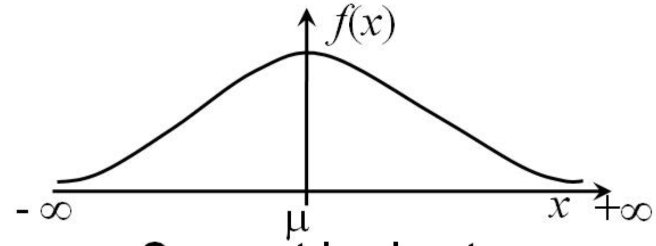
Modeling Techniques and Analysis

Lecture - 29

Course Instructor : Prof. P. P. MUJUMDAR

Department of Civil Engg., IISc.

Summary of the previous lecture



Symmetric about x = μ

- Normal distribution

$$f(x)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}\qquad -\infty<x<+\infty$$

$$Z=\frac{X-\mu}{\sigma}\qquad f(z)=\frac{1}{\sqrt{2\pi}}e^{-z^2/2}\qquad -\infty<z<+\infty$$

$X \sim N(\mu, \sigma^2)$

$F(z) = \int_{-\infty}^{z} f(z)\,dz = P[Z \le z]$

- Lognormal Distribution

μ_y, σ_y

$$f(x)=\frac{1}{\sqrt{2\pi}x\sigma_y}\mathrm{e}^{-(\ln x-\mu_y)^2/2\sigma_y^2}\qquad 0<x<\infty,\ 0<\mu_y<\infty,\ \sigma_y>0$$

X — LogNormal

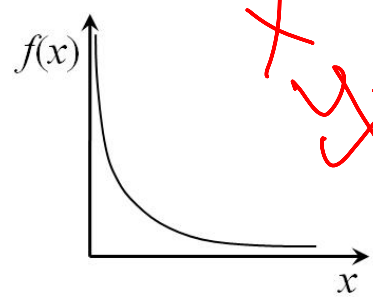
$Y = \ln X$ — Normal

- Exponential Distribution

$\lambda = 1/\mu$

$$f(x)=\lambda e^{-\lambda x}\qquad x>0,\ \lambda>0$$

$$F(x)=1-e^{-\lambda x}\qquad x>0,\ \lambda>0$$



CHANCE CONSTRAINED LINEAR PROGRAMMING (CCLP) FOR RESERVOIR DESIGN AND OPERATION

Chance Constrained LP

Capacity

Continuity

Deterministic LP model

Min K

Deterministic

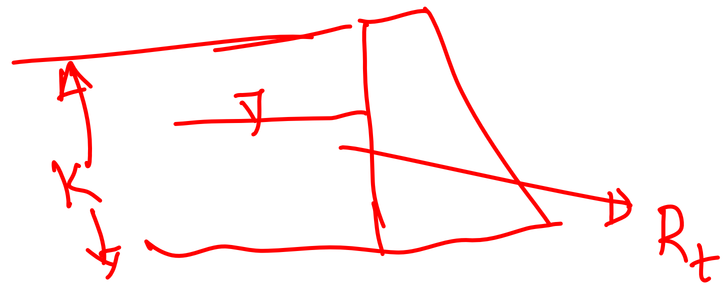
s.t. $S_{t+1} = S_t + Q_t - \cancel{E_t} - R_t$

$R_t \geq D_t$

$S_t \leq K$

$R_t \leq R_t^{\max}$

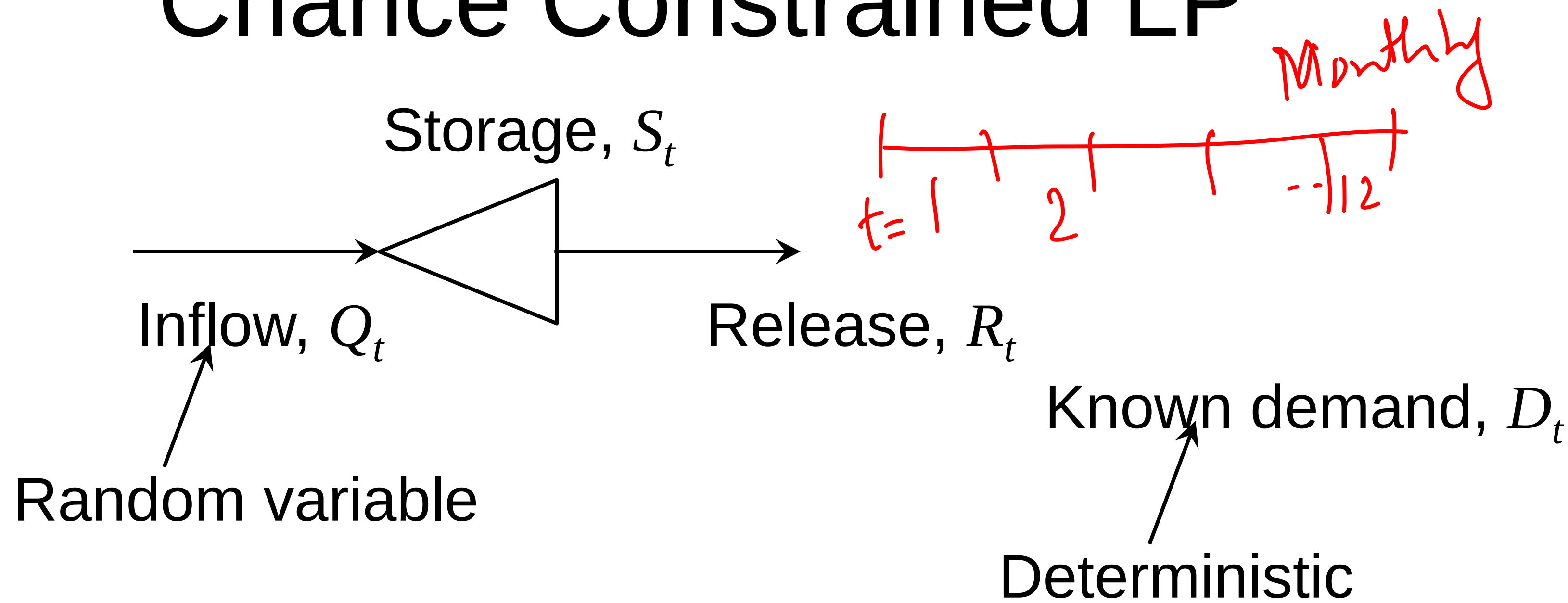
$S_t \geq S_{\min}$



Q_t, inflow during period, t, is a random variable.

Probability distribution of Q_t is known.

Chance Constrained LP



S_t and R_t being functions of Q_t, are also random in nature

- In a constraint containing two rvs, if the probability distribution of one is known, the probabilistic behavior of the second can be expressed as a measure of probability in terms of the probability of the first variable.

Chance Constrained LP

$R_t \geq D_t$

Deterministic

Chance constraint:

- The constraint relating release, R_t (random) and demand, D_t (deterministic) is expressed as a chance constraint.

Known

Known (specified)

$$P\left[R_t \geq D_t\right] \geq \alpha_1$$

Probability of release equaling or exceeding the known demand is at least equal to α_1 …. referred as reliability level.

- The reliability of meeting the demand in period t is at least α_1.

Chance Constrained LP

- Similarly, $P\left[R_t \leq R_t^{\max}\right] \geq \alpha_2$

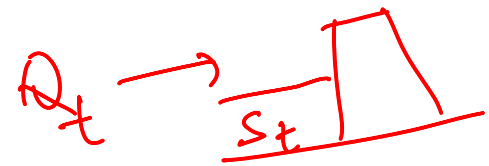
$$P\left[S_t \leq K\right] \geq \alpha_3$$

$$P\left[S_t \geq S_{\min}\right] \geq \alpha_4$$

Chance Constraints.

- Probability distribution of S_t and R_t to be determined from known probability distribution of Q_t.
- Since S_t, R_t and Q_t are interdependent, it is not possible to derive both probability distributions of S_t and R_t.
- To overcome this difficulty, Linear Decision Rule (LDR) is appropriately defined.

Chance Constrained LP



Linear Decision Rule (LDR):

- LDR relates the release, R_t, from the reservoir as a linear function of water available at period t.

$$R_t = S_t + Q_t - b_t$$

  b_t is a deterministic parameter (decision parameter).

- In this LDR, the entire amount, Q_t, is taken into account while making release decision.
- Depending on the proportion of inflow, Q_t, used in the LDR, a number of such LDRs may be formulated.

Chance Constrained LP

A general form of LDR may be written as

$$R_t = S_t + \beta_t Q_t - b_t \qquad 0 < \beta_t < 1 \quad \forall t$$

- $\beta_t = 0$ yields a relatively conservative release policy with release decisions related only to the storage.
- $\beta_t = 1$ yields an optimistic policy where the entire amount of water is available ($S_t + Q_t$), is used in the LDR.

Chance Constrained LP

Consider the LDR

$$R_t = S_t + Q_t - b_t$$

Neglecting evaporation,

Storage continuity equation is

$$S_{t+1} = S_t + Q_t - R_t$$

results in $S_{t+1} = b_t$

S_{t+1}, is set equal to b_t

- Treat S_t deterministic in the formulation.
- Advantage: other rv, R_t, may be expressed in terms of known distribution of Q_t. (Variance of Q_t is entirely transferred to the variance of R_t)

Chance Constrained LP

Deterministic constraint of a chance constraint:

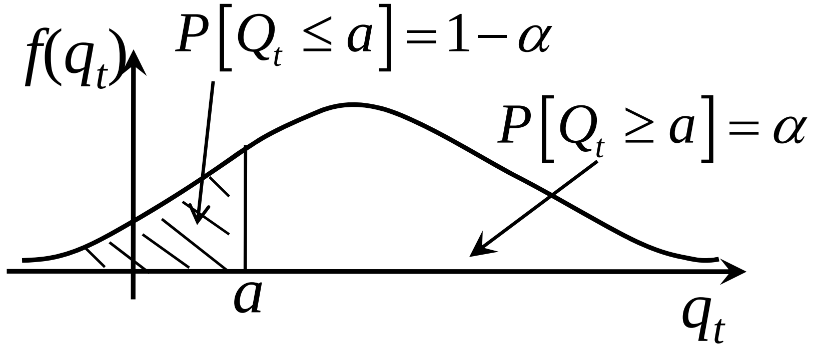
$$P\left[R_t \ge D_t\right] \ge \alpha_1$$

$$P\left[S_t + Q_t - b_t \ge D_t\right] \ge \alpha_1$$

$$P\left[b_{t-1} + Q_t - b_t \ge D_t\right] \ge \alpha_1$$

$$P\left[Q_t \ge D_t + b_t - b_{t-1}\right] \ge \alpha_1$$

$$P\left[Q_t \le \underbrace{D_t + b_t - b_{t-1}}\right] \le 1 - \alpha_1$$



$$P\left[Q_t \le a\right] + P\left[Q_t \ge a\right] = 1$$

$P[x \ge a] \ge \alpha_1$

$F(x) = P[x \le a]$

Deterministic with b_{t-1} and $b_t \rightarrow$ decision variables

and $D_t \rightarrow$ known demand

Chance Constrained LP

$$P\left[Q_t \le \underbrace{D_t + b_t - b_{t-1}}\right] \le 1-\alpha_1$$

$F_X(a) = P[X \le a]$

Equation rewritten as

$$F_{Q_t}\left(D_t + b_t - b_{t-1}\right) \le 1-\alpha_1$$

$F_{Q_t}\left(D_t + b_t - b_{t-1}\right)$ denotes CDF of the rv Q_t

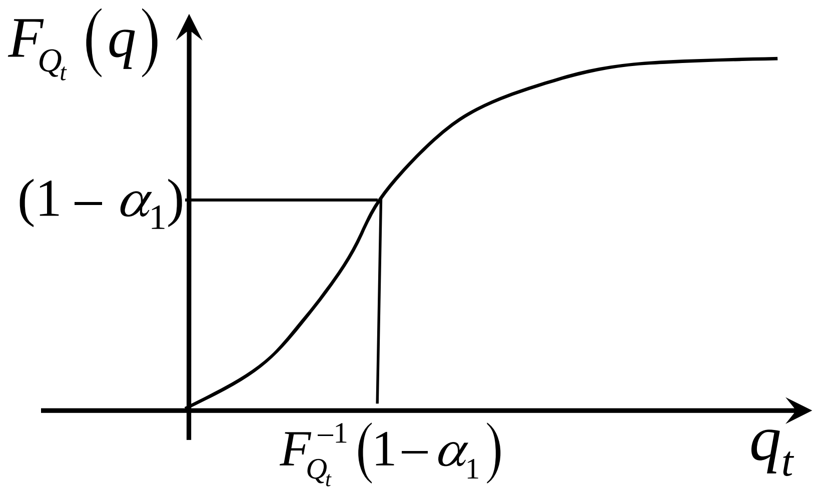
Deterministic equivalent of chance constraint $P\left[R_t \ge D_t\right] \ge \alpha_1$ is rewritten as

$$\left(D_t + b_t - b_{t-1}\right) \le F_{Q_t}^{-1}\left(1-\alpha_1\right)$$

Deterministic Equivalent

Chance Constrained LP

$F_{Q_t}^{-1}(1-\alpha_1)$ is the flow, q_t , at which the CDF value is $1 - \alpha_1$



Chance Constrained LP

The deterministic equivalent of a chance constraint, $P\left[R_t \le R_t^{\max}\right] \ge \alpha_2$, is similarly obtained as

$$\left(R_t^{\max} + b_t - b_{t-1}\right) \ge F_{Q_t}^{-1}\left(\alpha_2\right)$$

The other two constraints

$$\left.\begin{aligned} &P\left[S_t \le K\right] \ge \alpha_3 \\ &P\left[S_t \ge S_{\min}\right] \ge \alpha_4 \end{aligned}\right\}$$

Become deterministic constraints since the storage is set equal to the deterministic parameter b_{t-1}

Chance Constrained LP

The complete deterministic equivalent of CCLP is written as

~~Max~~ $\cancel{K}$ Min K

s.t.

$$\left.\begin{aligned}
&\left(D_t + b_t - b_{t-1}\right) \le F_{Q_t}^{-1}\left(1-\alpha_1\right)\\
&\left(R_t^{\max} + b_t - b_{t-1}\right) \ge F_{Q_t}^{-1}\left(\alpha_2\right)\\
&b_{t-1} \le K\\
&b_{t-1} \ge S_{\min}\\
&b_t \ge 0
\end{aligned}\right\} \quad \forall\, t$$

$$K \ge 0$$