INDIAN INSTITUTE OF SCIENCE

Water Resources Systems:
Modeling Techniques and Analysis

Lecture - 31

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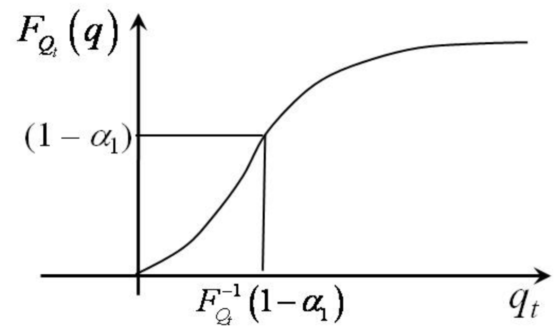
Summary of the previous lecture

- Deterministic equivalent of CCLP



$$
\begin{aligned}
\text{Min} \quad & K \\
\text{s.t.} \quad & \left(D_t + b_t - b_{t-1}\right) \le F_{Q_t}^{-1}\left(1-\alpha_1\right) \\
& \left(R_t^{\max} + b_t - b_{t-1}\right) \ge F_{Q_t}^{-1}\left(\alpha_2\right) \\
& b_{t-1} \le K \\
& b_{t-1} \ge S_{\min} \\
& b_t \ge 0
\end{aligned}
\quad \Bigg\} \; \forall\, t
$$

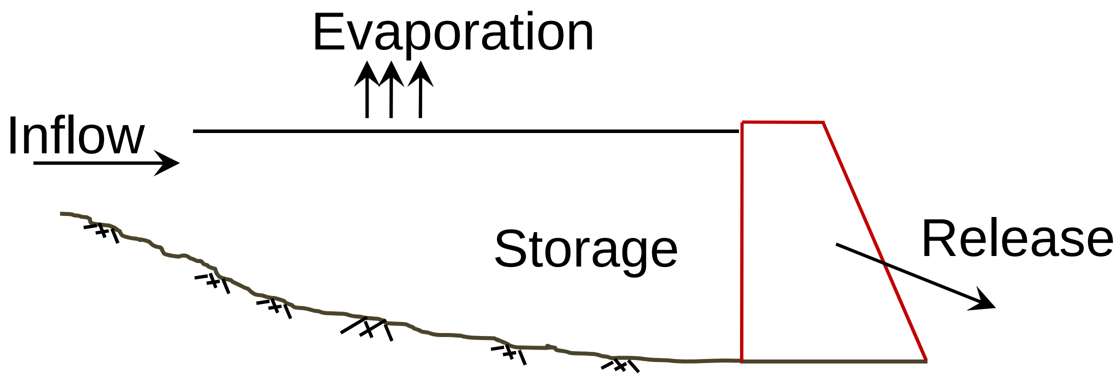
$$K \ge 0$$

$$
\begin{aligned}
\text{Min} \quad & K \\
\text{s.t.} \quad & P\left[R_t \ge D_t\right] \ge \alpha_1 \\
& P\left[R_t \le R_t^{\max}\right] \ge \alpha_2 \\
& P\left[S_t \le K\right] \ge \alpha_3 \\
& P\left[S_t \ge S_{\min}\right] \ge \alpha_4
\end{aligned}
$$

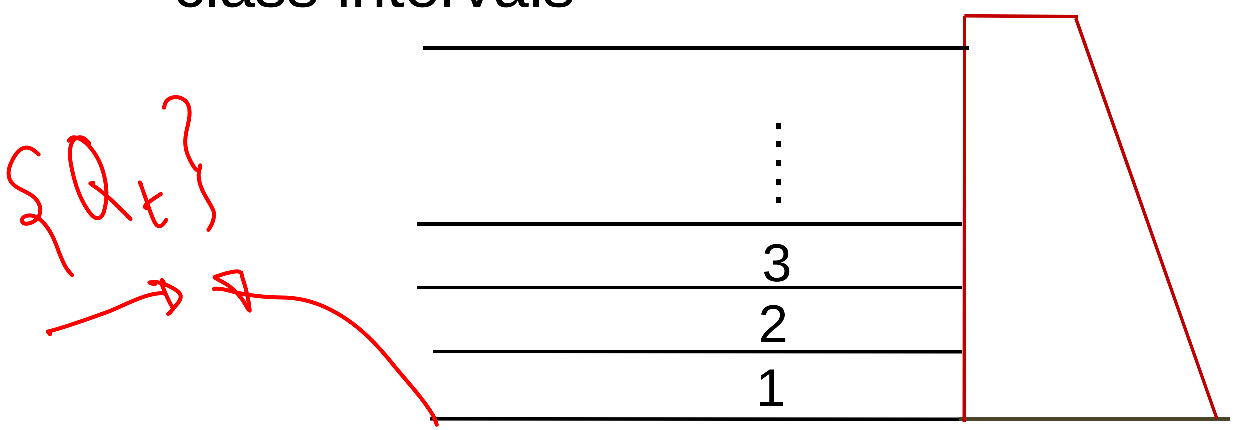
- Examples on writing the deterministic equivalent of CCLP

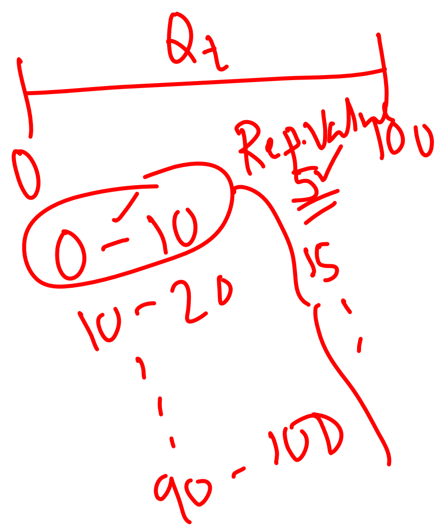
STOCHASTIC DYNAMIC PROGRAMMING (SDP)

Stochastic Dynamic Programming



- Reservoir storage and inflow divided into discrete class intervals





- Inflow - Stochastic process.

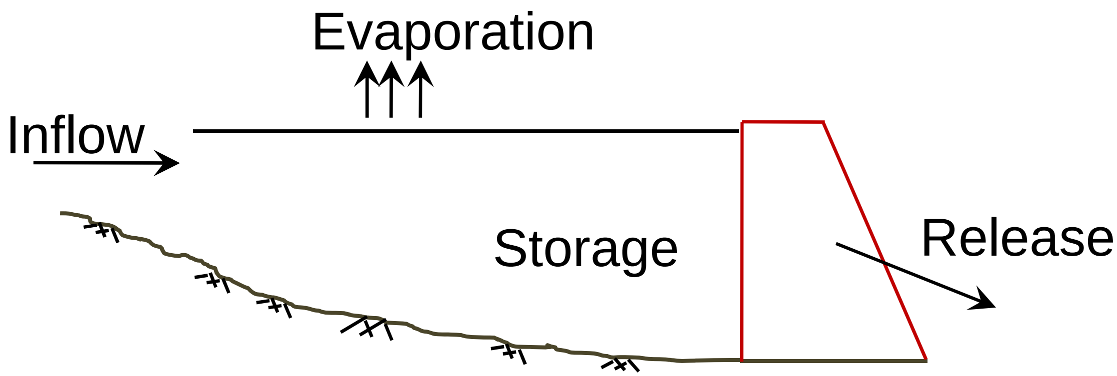
Stochastic Dynamic Programming

- Stage: Time period (e.g., month, week etc.) at which decisions need to be taken.
- State variables: storage at the beginning of period t and inflow during period t.
- Decision: Release from the reservoir during period t.
- Objective: To optimize (maximize or minimize) the expected value of a system performance measure.

  For example,

  - Maximize the expected value of annual hydropower generated
  - Minimize the expected value of annual flood damage.

Stochastic Dynamic Programming



- System performance measure is a function of release and/or storage
  - Hydropower generated
  - Crop yield achieved
  - Monitory benefits
  - Magnitude of flood mitigation etc.

Stochastic Dynamic Programming

Notation:

	t		$t+1$	
k	i	l	j	Class intervals
S_{kt}	Q_{it}	$S_{l\,t+1}$	Q_{jt+1}	Representative values

Q : Inflow S : Storage R : Release

Loucks et al (1981)

Vedula and Mujumdar (2005)

k : Class interval of storage at the beginning of period t

i : Class interval of inflow during period t

l : Class interval of storage at the beginning of period $t+1$

j : Class interval of inflow during period $t+1$

Stochastic Dynamic Programming

	t		$t+1$	
k		l		Storage class interval
	i		j	Inflow class interval
S_{kt}		$S_{l\ t+1}$		Storage representative values
	Q_{it}		Q_{jt+1}	Inflow representative values

All variables are discretised into a no. of class intervals

Stochastic Dynamic Programming

- First order Markov chain (or single step Markov chain):

$$P\left[X_t / X_{t-1}, X_{t-2}, ...X_0\right] = P\left[X_t / X_{t-1}\right]$$

X_t : Random variable

- Inflows during time intervals ranging from 10 days to a year may be assumed to follow a single step Markov chain.

- Transition probabilities are used to measure the dependence of the inflow during period t+1 on the inflow during the period t.

Stochastic Dynamic Programming

- The transition probability P_{ij}^{t} is defined as the probability that the inflow during period $t+1$ will be in class interval j, given that the inflow during the period t lies in the class interval i,

$$P_{ij}^{t} = P\left[Q_{t+1} = j / Q_t = i\right]$$

where $Q_t = i$ indicates that the inflow during the period t belongs to the discrete class interval i.

- The transition probabilities are estimated from historical inflow data.

Stochastic Dynamic Programming

- Release during the period t is R_{kilt}
- Storage state changes because of inflow, release and evaporation.
- Inflow state changes randomly from period to period.

Stochastic Dynamic Programming

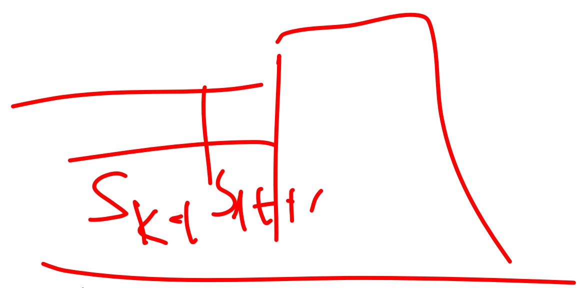
State transformation:

$$S_{l\,t+1} = S_{k\,t} + Q_{k\,t} - E_{kl\,t} - R_{kil\,t}$$

E_{klt} is evaporation loss during period t corresponding to storage class intervals k and l.

- For given k, i and l, release is computed as

$$R_{kil\,t} = S_{k\,t} + Q_{k\,t} - E_{kl\,t} - S_{l\,t+1}$$



Stochastic Dynamic Programming

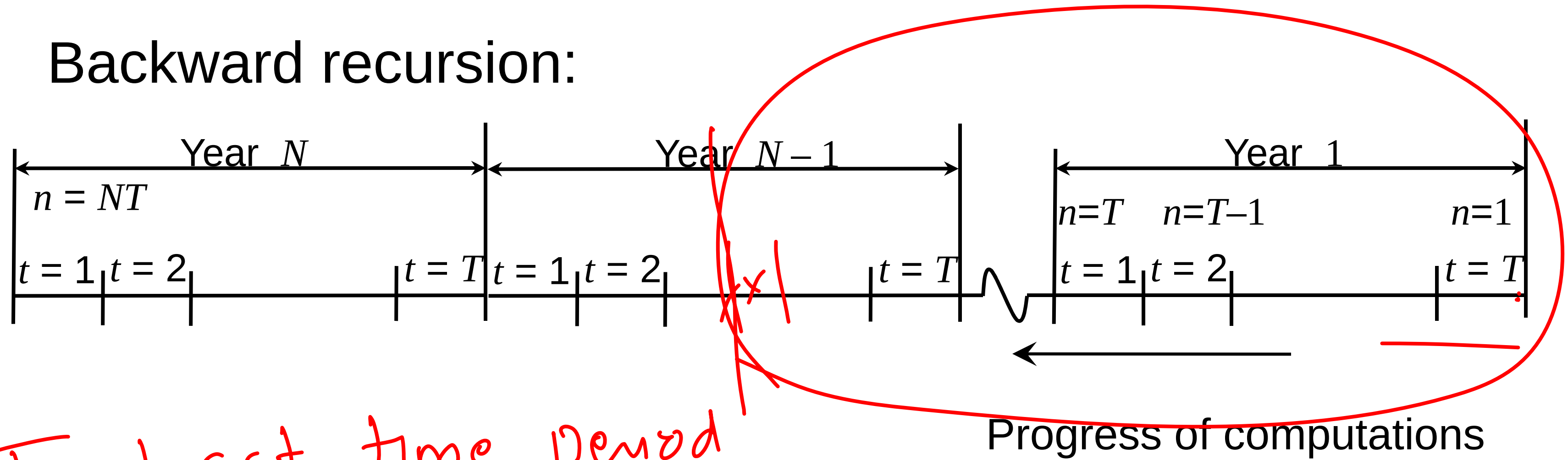
- System performance measure: B_{kilt}
- B_{kilt} is, in general, a function of S_{kt}, R_{kilt} and S_{lt+1}

  For example,

  - Amount of power generated during period t.
  - Deficit release from target in period t.

Stochastic Dynamic Programming

Backward recursion:



T: Last time period in a year;

$f_t^n(k,i)$: Optimum expected value of the system performance measure at stage n, corresponding to time period is t.