INDIAN INSTITUTE OF SCIENCE

Water Resources Systems:
Modeling Techniques and Analysis

Lecture - 32

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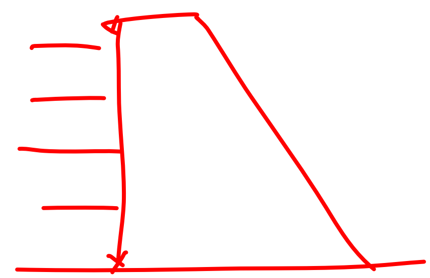
Summary of the previous lecture

- Stochastic dynamic programming

	t		$t+1$	
k	i	l	j	Class intervals
S_{kt}	Q_{it}	$S_{l\,t+1}$	Q_{jt+1}	Representative values

- State transformation

$$S_{l\,t+1} = S_{k\,t} + Q_{i\,t} - E_{kl\,t} - R_{kil\,t}$$

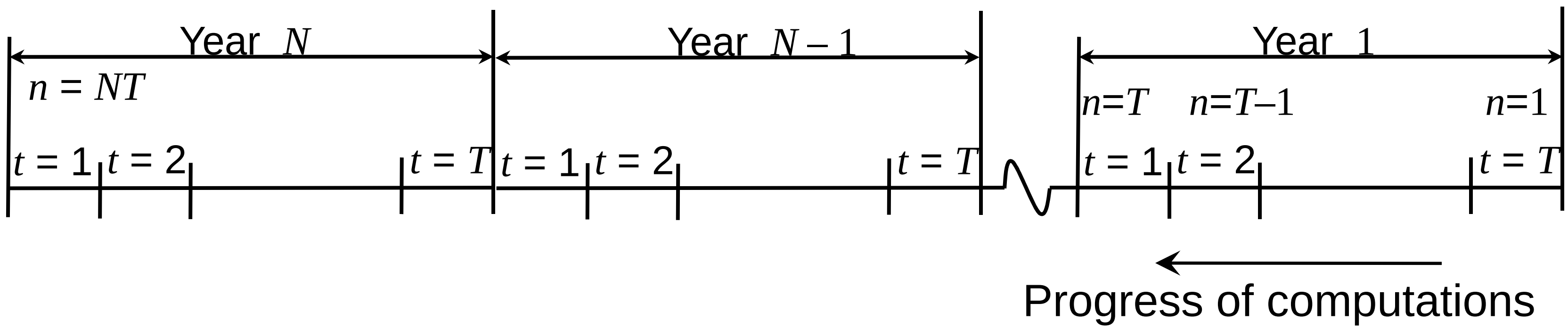


trapping state?

100 200

150 ← Representative

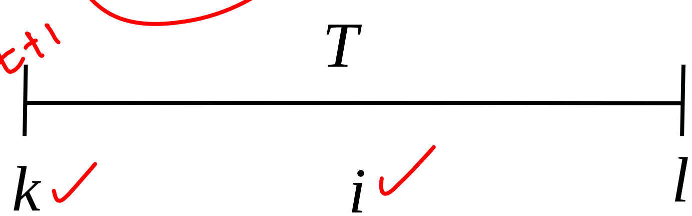
Stochastic Dynamic Programming



$n = 1$ and $t = T$: $- E_{klt}$

$p_{ij}^t = P[Q_{t+1} = j \mid Q_t = i]$

$R_{kilt} = S_{kt} + Q_{it} - S_{l,t+1}$

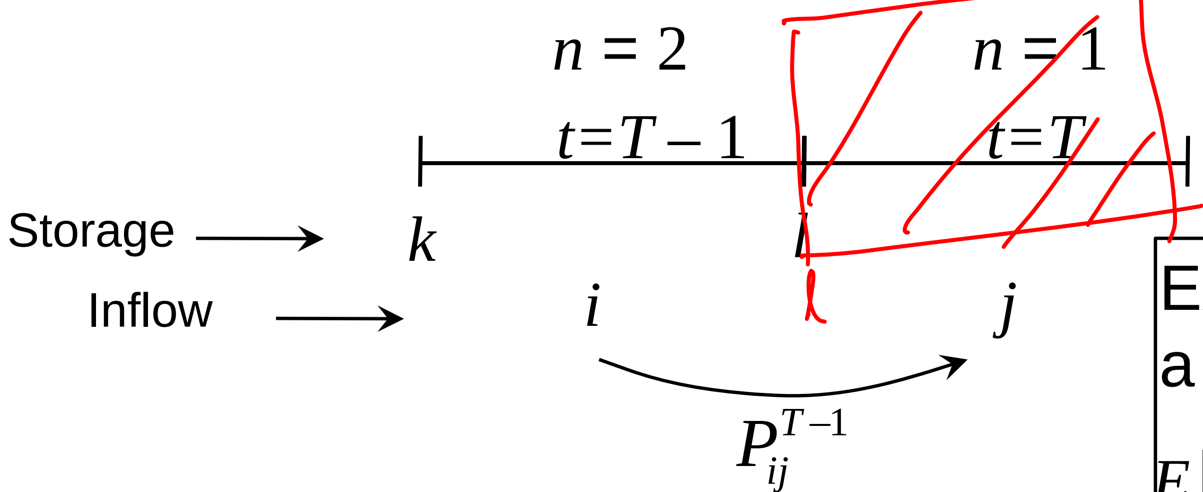


$$f_T^1(k,i) = \underset{\{feasible\ l\}}{Max}\left[B_{kilt}\right] \qquad \forall k,i$$

For a given k and i, only those values of l are feasible that result in a non-negative value of release, R_{kilt}

Stochastic Dynamic Programming

$n = 2$ and $t = T - 1$:



Expected value of a discrete rv X,

$$E[X] = \sum_m x_m p(x_m)$$

l : End of period storage for T–1

: Beginning of period storage for T

$$f_{T-1}^2(k,i) = \underset{\{feasible\ l\}}{Max} \left[B_{kil\,T-1} + \sum_j P_{ij}^{T-1} f_T^1(l,j) \right] \qquad \forall\, k,i$$

Stochastic Dynamic Programming

General recursive relationship:

$$f_t^n(k,i) = \underset{\{feasible\ l\}}{Max} \left[B_{kil\,t} + \sum_j P_{ij}^t f_{t+1}^{n-1}(l,j) \right]$$

For current period t

Expected value of system performance measure up to the previous stage n–1.

Inflow transition probabilities are assumed to remain the same from year to year …. Stationary stochastic process

Stochastic Dynamic Programming

t=2 t=1 t=2 t=1

Steady state policy:

$$f_t^{\,n}(k,i) = \underset{\{feasible\ l\}}{Max}\left[B_{kilt} + \sum_j P_{ij}^t\, f_{t+1}^{\,n-1}(l,j) \right]$$

Solution of this equation recursively converge to a steady state solution.

$$\underbrace{f_t^{\,n+T}(k,i) - f_t^{\,n}(k,i)}_{\text{Expected annual performance}}$$

Remains constant $\forall\ k$, i and t .

Optimal steady state policy: $l^*(k,i,t)$ $\forall\ k$, i and t .

End of period storage for known initial storage k and inflow i in period t.

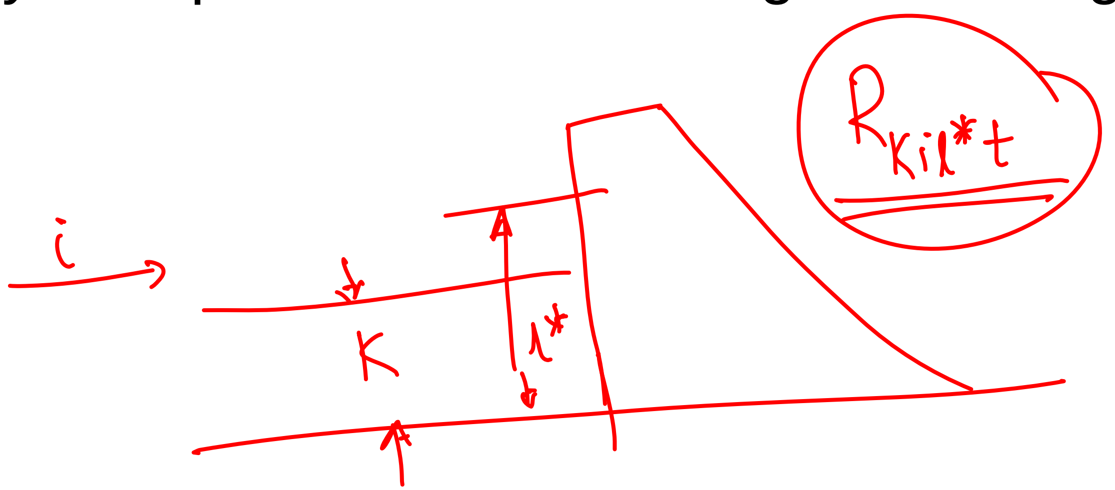
Stochastic Dynamic Programming

Steady state policy:

- Optimal steady state policy $l^*(k,i,t)$ remains unaltered.

E_{klt}

- Annual system performance converges to a single value.



Example – 1

Lecture on Markov Chains

NPTEL Course Stochastic Hydrology

Obtain the steady state policy with an objective to minimize the expected value of the sum of the square of deviations of release and storage from their respective targets, over a year with two periods. Neglect the evaporation loss. If the release is greater than release target, the deviation is set to zero. The data is as follows.

Period $t = 1$

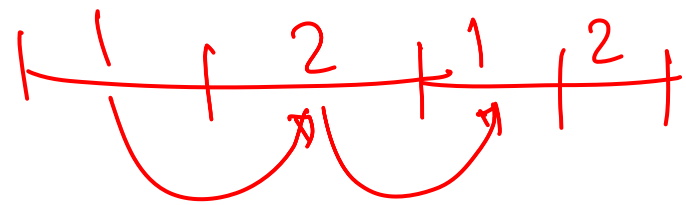
i	Q_i^t	k	S_k^t
1	15	1	30
2	25	2	40

Period $t = 2$

i	Q_i^t	k	S_k^t
1	35	1	20
2	45	2	30

Example – 1 (Contd.)

Target storage T_s = 30
Target release T_r = 30



Inflow transition probabilities:

	$t = 2$		
	j		
$t = 1$	i	1	2
	1	0.5	0.5
	2	0.3	0.7

	$t = 1$		
	j		
$t = 2$	i	1	2
	1	0.4	0.6
	2	0.8	0.2

Example – 1 (Contd.)

Solution:

The system performance measure, B_{kilt}, is the sum of the square of deviations of release and storage from their respective targets

$$B_{kilt} = \left(R_{kilt} - T_r\right)^2 + \left(S_k^t - T_s\right)^2$$

Target storage T_s = 30
Target release T_r = 30

The system performance measure, B_{kilt}, is tabulated $\forall$ k,i,l and t.

Example – 1 (Contd.)

Period t = 1

$R_{kilt} = S_k^t + Q_i^t - S_l^{t+1}$

30 30

k	S_k^t	i	Q_i^t	l	S_l^{t+1}	R_{kilt}	$(S_k^t - T_s)^2$	$(R_{kilt} - T_r)^2$	B_{kilt}
1	30	1	15	1	20	25	0	25	25
1	30	1	15	2	30	15	0	225	225
1	30	2	25	1	20	35	0	0	0
1	30	2	25	2	30	25	0	25	25
2	40	1	15	1	20	35	100	0	100
2	40	1	15	2	30	25	100	25	125
2	40	2	25	1	20	45	100	0	100
2	40	2	25	2	30	35	100	0	100

Example – 1 (Contd.)

Period $t = 2$

k	$S_k^{\,t}$	i	$Q_i^{\,t}$	l	$S_l^{\,t+1}$	R_{kilt}	$(S_k^{\,t}-T_s)^2$	$(R_{kilt}-T_r)^2$	B_{kilt}
1	20	1	35	1	30	25	100	25	125
1	20	1	35	2	40	15	100	225	325
1	20	2	45	1	30	35	100	0	100
1	20	2	45	2	40	25	100	25	125
2	30	1	35	1	30	35	0	0	0
2	30	1	35	2	40	25	0	25	25
2	30	2	45	1	30	45	0	0	0
2	30	2	45	2	40	35	0	0	0