INDIAN INSTITUTE OF SCIENCE

Water Resources Systems:

Modeling Techniques and Analysis

Lecture - 34

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Summary of the previous lecture

- Stochastic dynamic programming

Steady state probabilities:

$$PR_{ljt+1} = \sum_k \sum_i PR_{kit} P_{ij}^t \qquad \forall\ l, j \text{ and } t$$

$$l = l^*(k,i,t)$$

$$\sum_k \sum_i PR_{kit} = 1 \qquad \forall\ t$$

$$PS_{kt} = \sum_i PR_{kit} \qquad \forall\ k, t$$

$$PQ_{it} = \sum_k PR_{kit} \qquad \forall\ i, t$$

FUZZY OPTIMIZATION

Fuzzy Optimization

General optimization problem:

Maximize or Minimize $f(X)$

s.t.

$g_j(X) \leq 0 \qquad j = 1, 2, \ldots\ldots m$

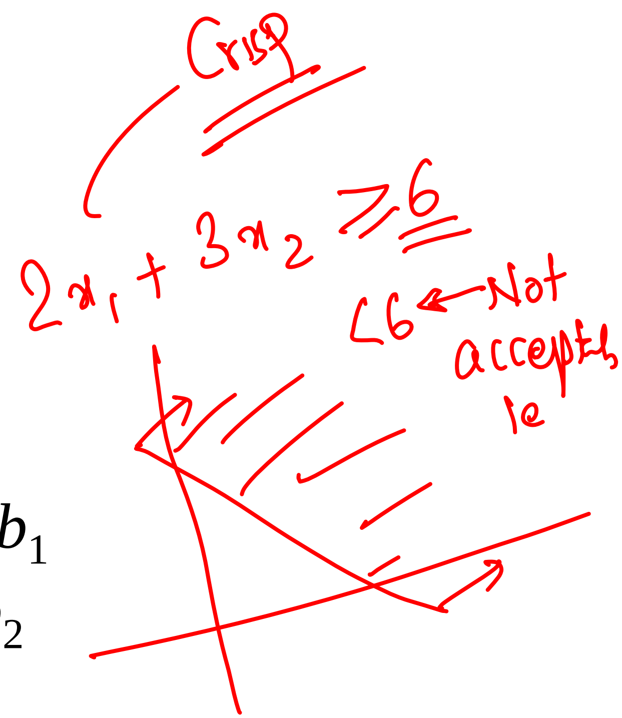
Decision Vector, $X = (x_1, x_2, x_3, \ldots\ldots x_n)$

$x_1, x_2, x_3, \ldots\ldots x_n$: Decision Variables

Fuzzy Optimization

Linear programming

- Linear objective function
- Linear constraints
- Non-negative decision variables



e.g.,:

MAX. $c_1x_1 + c_2x_2 + \ldots + c_nx_n$

s.t.

$$a_{11}x_1 + a_{12}x_2 + \ldots\ldots + a_{1n}x_n \leq b_1$$
$$a_{21}x_1 + a_{22}x_2 + \ldots\ldots + a_{2n}x_n \leq b_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots + a_{mn}x_n \leq b_m$$

$$x_1 \geq 0;\ x_2 \geq 0; \ldots\ldots x_n \geq 0.$$

Fuzzy Optimization

Fuzzy optimization

- Objective function and/or constraints may be fuzzy.

  e.g., Minimized cost of a process design should be about z_0 or less

  $X \geq 5$ ….. Crisp constraint

  X ~> 5 ….. Fuzzy constraint

  X is about 5 or greater, which means a solution

  $X < 5$ is also acceptable, but to a lesser degree

- Fuzzy goals and constraints
  - Reflect degree of satisfaction of decision makers

Fuzzy Optimization

Fuzzy decision

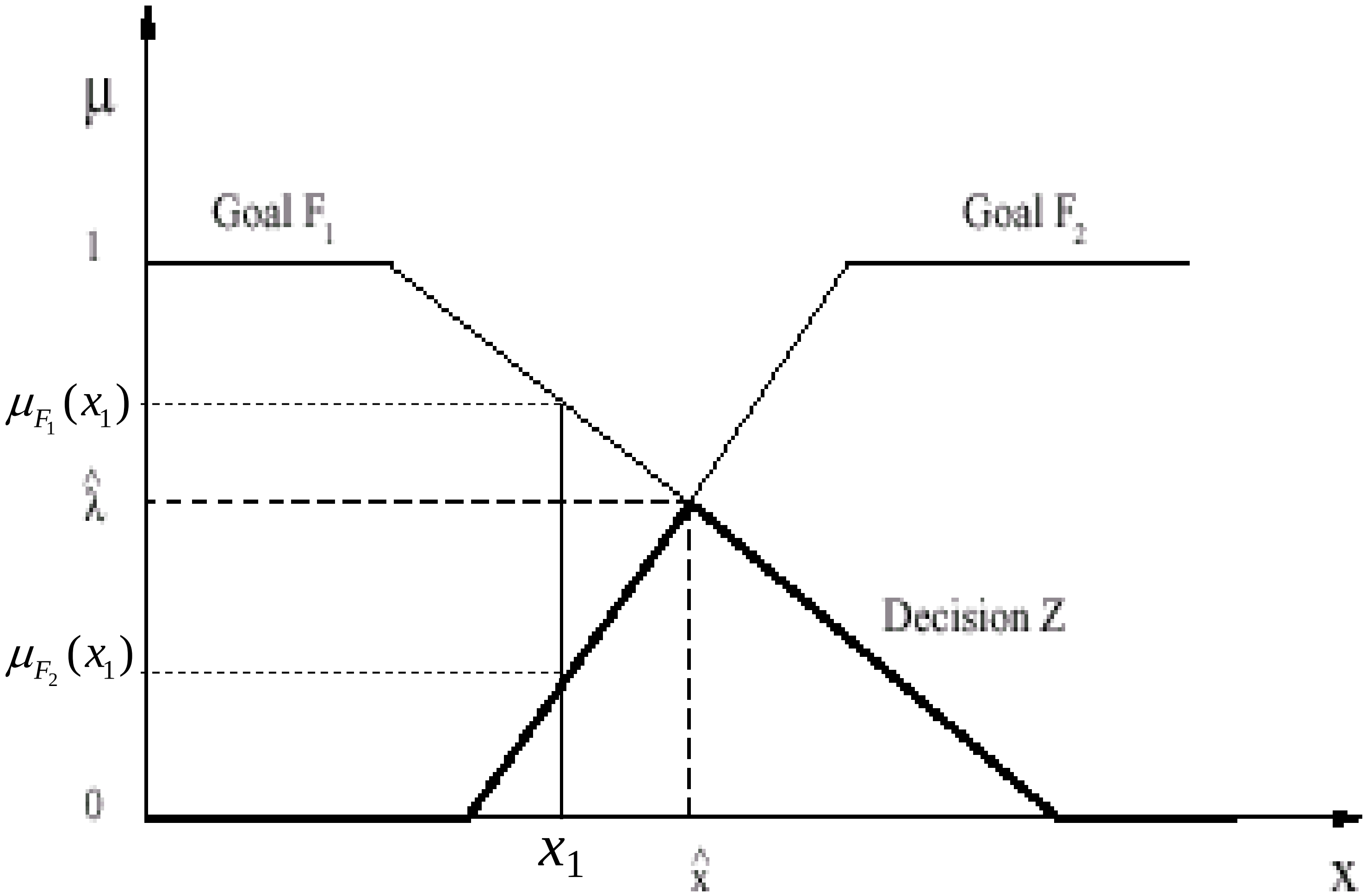
- Confluence of "fuzzy objectives" and "fuzzy constraints" defined as "fuzzy decision" (Bellman and Zadeh (1970)); represented as fuzzy sets

$$Z = F_1 \bigcap F_2$$

$$\mu_Z(x) = \lambda = \min\left[\mu_{F_1}(x), \mu_{F_2}(x)\right]$$

$$\mu_Z\left(\hat{x}\right) = \hat{\lambda} = \max_{X \in Z}\left[\mu_Z\left(x\right)\right]$$

Fuzzy Optimization



Concept of fuzzy decision

Fuzzy Optimization

e.g.,

X= [0,∞] ….. Set of alternatives

$$\mu_c(x) = 0 \qquad x \le 10$$

$$= 1 - \left[1 + 0.1\left(x - 10\right)^2\right]^{-1} \qquad x > 10$$

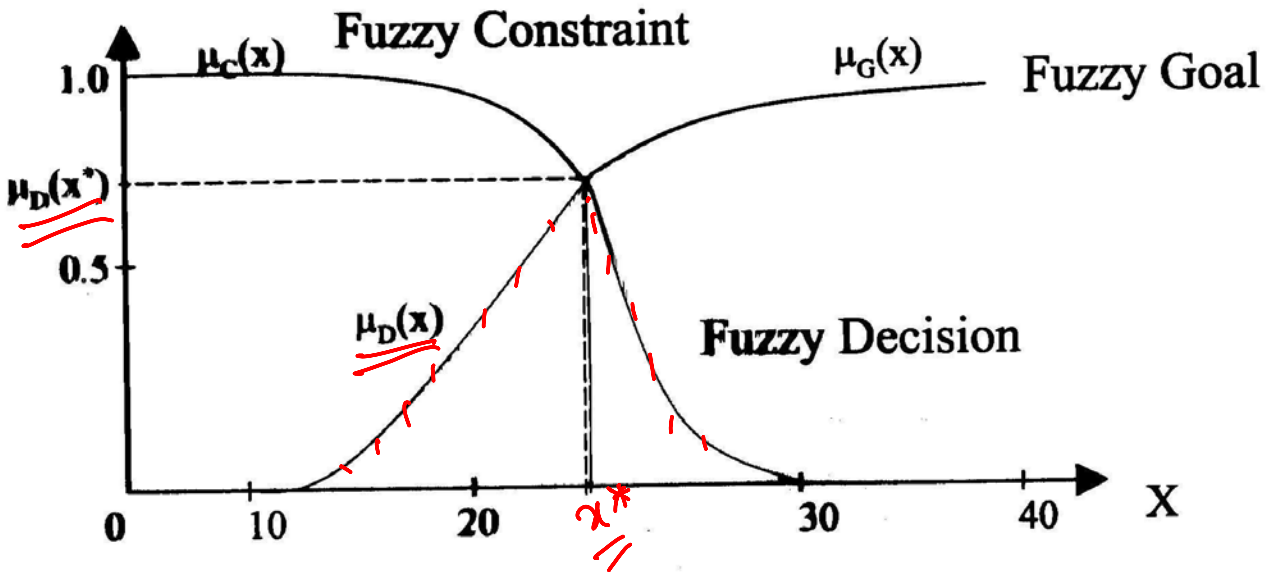
Fuzzy goal: To make x sufficiently larger than 10.

$$\mu_c(x) = 0 \qquad x \ge 30$$

$$= \left[1 + x\left(x - 30\right)^{-2}\right]^{-1} \qquad x < 30$$

Fuzzy constraint: x should be a lot smaller than 30.

Fuzzy Optimization



$\mu_D(x)$: Membership Function of the Fuzzy Decision
= Min $[\mu_G(x), \mu_C(x)]$

x^*, Optimal Value of x, $\mu_D(x^*) = \lambda^* = \max\ [\mu_D(x)]$

- In Practical Situations a Large Number of Fuzzy Goals, Fuzzy Constraints and Crisp Constraints Exist.

Fuzzy Optimization

Conventional LP:

$$\text{Min } Z = [C][X]$$

s.t.

$$[A][X] \leq [b]$$

$$[X] \geq [0]$$

$[C] = (c_1, c_2, \ldots\ldots c_n)$ …..1x n

$[X] = (x_1, x_2, \ldots\ldots x_n)^T$ …..n x 1

$[A] = [a_{ij}]$ ….m x n

$[b] = (b_1, b_2, \ldots\ldots b_n)^T$ …..m x 1

n Variables, m Constraints.

Z should be about 80 or less

$[A][X] \lesssim [b]$

Fuzzy Optimization

With Fuzzy Goals And Constraints,

$[C][X] \sim< z_0$ …… Desired Min Value for the OF.

$[A][X] \sim< [b]$ ……Desired Values for the RHS of Constraints

Since Fuzzy Goals and Constraints play same role in Fuzzy Decision Making, FLP is written as,

$$[B][X] \sim< [b']$$

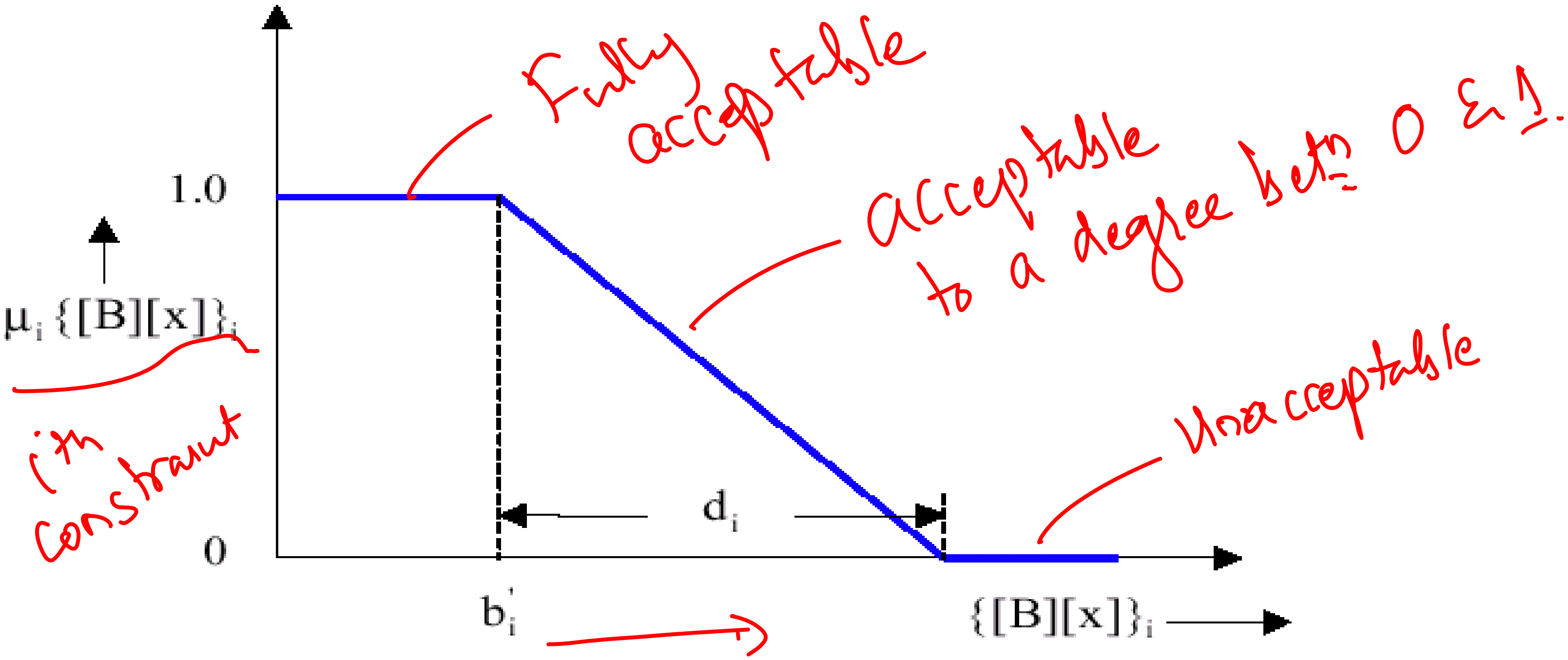
Fuzzy Constraint

$$[X] \geq [0]$$

With

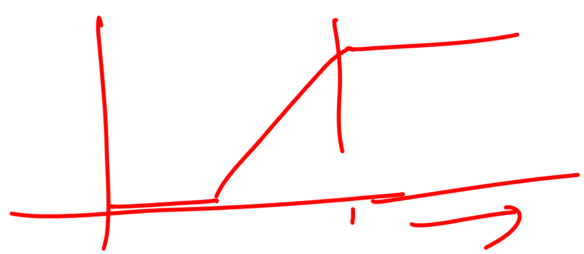
$$[B] = \begin{bmatrix} [C] \\ [A] \end{bmatrix} \qquad [b'] = \begin{bmatrix} [z_0] \\ [b] \end{bmatrix}$$

Fuzzy Optimization



$$[B][X] \sim< [b']$$

$$[X] \geq [0]$$



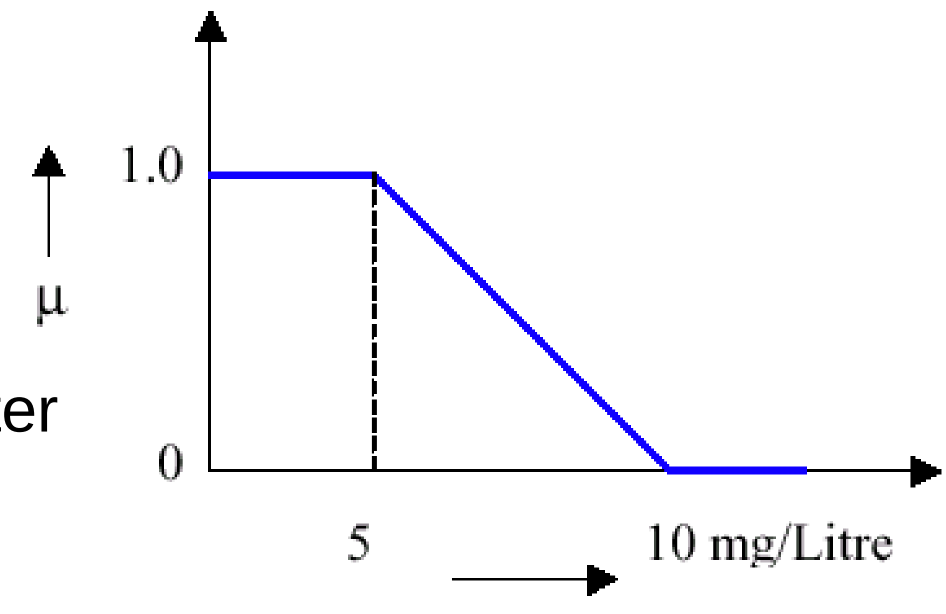
Fuzzy Optimization

Linear membership for i^{th} constraint

Linear function that is 1 when the i^{th} constraint is fully satisfied, 0 when it is not satisfied by a width d_i or greater and values between 0 and 1 for intermediate cases

d_i : Value determined subjectively by decision-maker.

e.g., BOD level in water treatment.

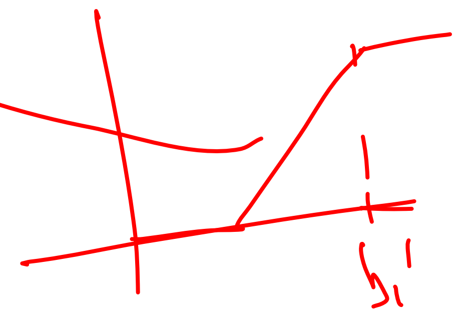


Fuzzy Optimization

$$\mu_i\left[(Bx)_i\right] = \begin{cases} 1 & \text{....} \quad (Bx)_i \le b_i' \\ 1 - \dfrac{\left[(Bx)_i - b_i'\right]}{d_i} & \text{....} \quad b_i' \le (Bx)_i \le \left(b_i' + d_i\right) \\ 0 & \text{....} \quad (Bx)_i \ge \left(b_i' + d_i\right) \end{cases}$$

From Fuzzy Decision concept,

$$\mu_D\left(x^*\right) = \underset{x \ge 0}{Max}\left[\underset{1 \le i \le m}{Min}\left\{\mu_i\left[(Bx)_i\right]\right\}\right]$$



Fuzzy Optimization

$$b_i'' = \frac{b_i'}{d_i}; \quad \left(B' x\right)_i = \frac{(Bx)_i}{d_i}$$

$$\mu_D(x^*) = \underset{x \geq 0}{Max}\left[\underset{1 \leq i \leq m}{Min}\left\{\mu_i\left[\left(Bx\right)_i\right]\right\}\right]$$

$$= \underset{x \geq 0}{Max}\left[\underset{1 \leq i \leq m}{Min}\left\{1 + b_i'' - \left[\left(B' x\right)_i\right]\right\}\right]$$

Max λ

s.t. $1 + b_i'' - \left(B' x\right)_i \geq \lambda \quad \forall\, i$

$x_i \geq 0 \quad \forall\, i$

Crisp equivalent of Fuzzy LP.