INDIAN INSTITUTE OF SCIENCE

Water Resources Systems:

Modeling Techniques and Analysis

Lecture - 36

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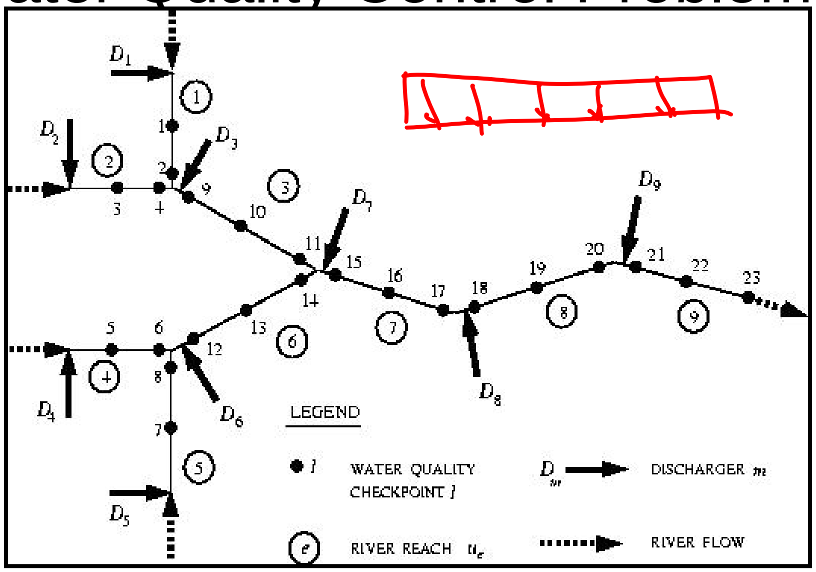
Summary of the previous lecture

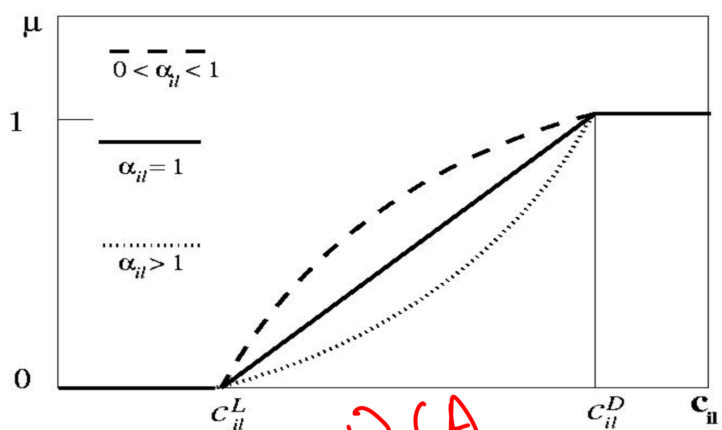
Stream Water Quality Control Problems



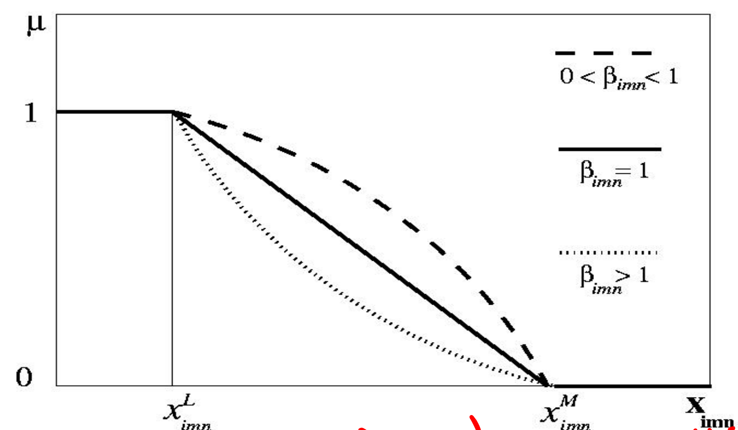
Non-reactive pollutant

x : fraction removal level

i, m, n



PCA



Discharges

Fuzzy Optimization

- The concentration level, C_{il}, of the water quality parameter i at the checkpoint l can be related to the fraction removal level, x_{imn}, of the pollutant *n* from the discharger m to control the water quality parameter i, though the transfer function that may be mathematically expressed as

$$C_{il} = \sum_{m=1}^{N_d} \sum_{n=1}^{N_p} f_{ilmn}(L_{ilmn}, x_{imn}) + \sum_{p=1}^{N_t} \sum_{n=1}^{N_p} f_{ilpn}(L_{ilpn})$$

where L_{ilmn} is the concentration of the pollutant n prior to treatment from the discharger m that affects the water quality parameter i at the checkpoint l,

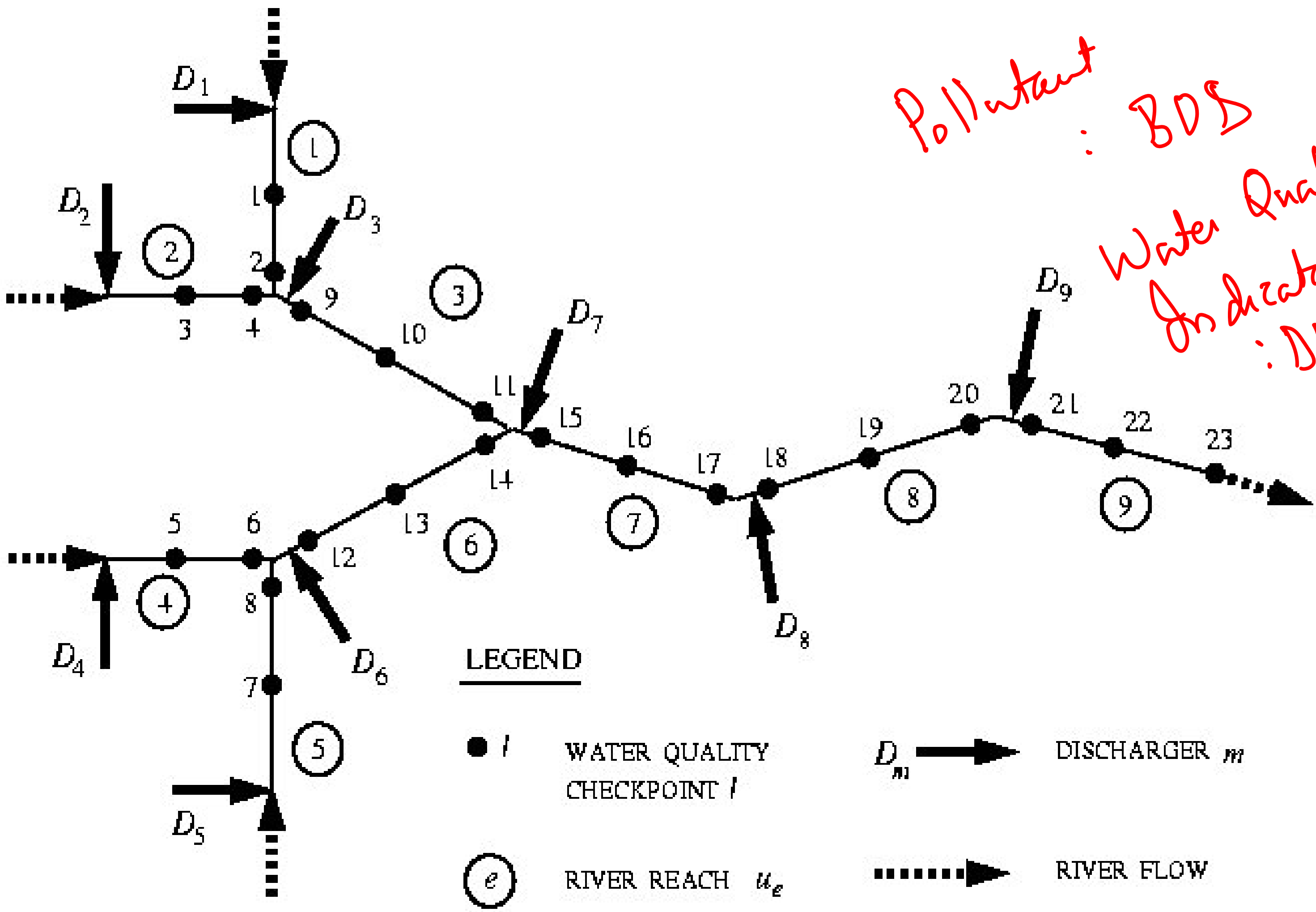
L_{ilpn} is the concentration of the pollutant n from the uncontrollable source p that affects the water quality parameter i at the checkpoint l.

Fuzzy Optimization

- The transfer functions $f_{ilmn}(\cdot, \cdot)$ and $f_{ilpn}(\cdot)$ represent the concentration levels of the water quality parameter i due to $L_{ilmn}(1 - x_{imn})$, and L_{ilpn} respectively
- These transfer functions can be evaluated using appropriate mathematical models that determine the spatial and temporal distribution of the water quality parameter due to the pollutants in the river system
- The solution of the optimization model are X^* and λ^* where X^* is the set of optimal fraction removal levels, and λ^* is the maximized m

Example – 2

Consider a hypothetical river network as shown



Pollutant : BOD
Water Quality Indicator : DO-deficit,

Example – 2 (Contd.)

- The river network is discretized into 9 river reaches.
- Each reach receives a point-source of BOD load from a discharger located at the beginning of the reach.
- The only pollutant in the system is the point source of BOD waste load.
- Water quality parameter of interest is the dissolved oxygen deficit (DO deficit) at 23 number of checkpoints due to the point-sources of BOD.
- The data pertaining to the river flows and effluent flows are given in table

Effluent Flow Data				River Flow Data								
Discharger (1)	Effluent Flow Rate (10^4 m^3/day) (2)	BOD Concentration (mg/L) (3)	Do Concentration (mg/L) (4)	River Reach r_e (5)	Flow (10^6 m^3/day) (6)	Total Flow (10^6 m^3/day) (7)	Time Of Flow (day) (8)	Deoxygenation Rate Constant (1/day) (9)	Reaeration Rate Constant (1/day) (10)	Saturation Do Conc. (mg/L) (11)	Permissible DO Deficit (mg/L) (12)	Desirable Do Deficit (mg/L) (13)
D_1	2.134	1250	1.23	r_1	4.6183	4.63964	0.316	0.331	0.847	10.10	3.5	0.0
D_2	10.738	525	2.15	r_2	3.2574	3.36478	1.312	0.328	0.743	9.85	3.0	0.5
D_3	4.178	1878	2.16	r_3	7.8757	8.04620	0.642	0.378	0.532	9.64	3.5	0.0
D_4	6.415	723	1.80	r_4	3.9821	4.04625	1.281	0.410	0.831	9.78	3.5	0.0
D_5	8.319	1272	2.40	r_5	5.2394	5.32259	0.732	0.320	0.754	10.20	3.0	0.0
D_6	7.554	2080	1.41	r_6	9.2215	9.44438	1.218	0.357	0.670	9.90	3.0	0.5
D_7	9.832	2564	1.62	r_7	17.0972	17.5889	1.787	0.393	0.580	9.85	4.0	1.0
D_8	3.511	1842	1.70	r_8	17.0972	17.624	1.823	0.383	0.425	9.65	3.5	1.5
D_9	5.180	932	1.93	r_9	17.0972	17.6758	2.131	0.390	0.210	9.50	4.0	1.5

Example – 2 (Contd.)

- The transfer function that expresses the DO deficit at a checkpoint in terms of the concentration of point-source of BOD and the fraction removal levels can be obtained using the one dimensional steady state Streeter-Phelps BOD-DO equations*.

* Chapra, S.C., Surface water-quality modeling, The Mc-Graw Hill Companies Inc. 1997

Sasikumar, K., and Mujumdar, P. P., (1998) Fuzzy optimization model for water quality management of a river system, *Journal of Water Resources Planning and Management*, 124(2), 79-88.

Subimal Ghosh, H. R. Suresh and P. P. Mujumdar (2008), Fuzzy Waste Load Allocation: Application to a Case Study, *Journal of Intelligent Systems*, 17(1-3), 283-296.

Example – 2 (Contd.)

Solution:

- Since only one pollutant and one water quality parameter are considered, the suffixes i and n are dropped from the constraints and OF for convenience.
- Denote the DO deficit at the water quality checkpoint l by C_l, and the fraction removal level for the m^{th} discharger by x_m.

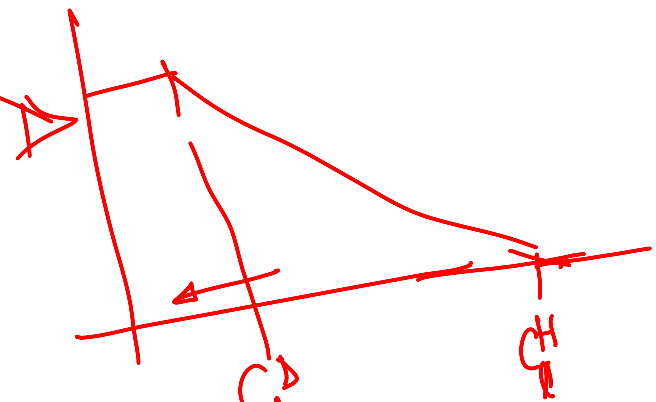
Example – 2 (Contd.)

Using linear membership functions for the fuzzy goals (i.e., $\alpha_{(..)} = \beta_{(...)} = 1$).

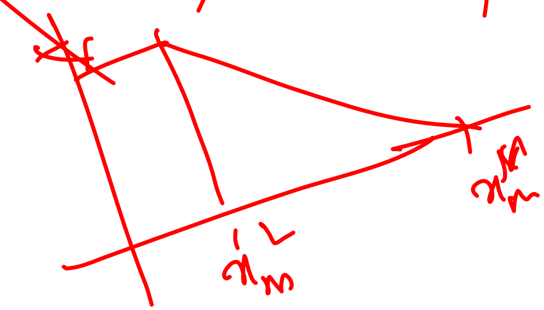
$$\max \quad \lambda$$

s.t.

$$\frac{C_l^H - C_l}{C_l^H - C_l^D} \geq \lambda \qquad \forall\, l$$



$$\frac{x_m^M - x_m}{x_m^M - x_m^L} \geq \lambda \qquad \forall\, m$$



$$c_l^D \leq c_l \leq c_l^H \qquad \forall\, l$$

$$x_m^L \leq x_m \leq x_m^M \qquad \forall\, m$$

$$0 \leq \lambda \leq 1$$

Example – 2 (Contd.)

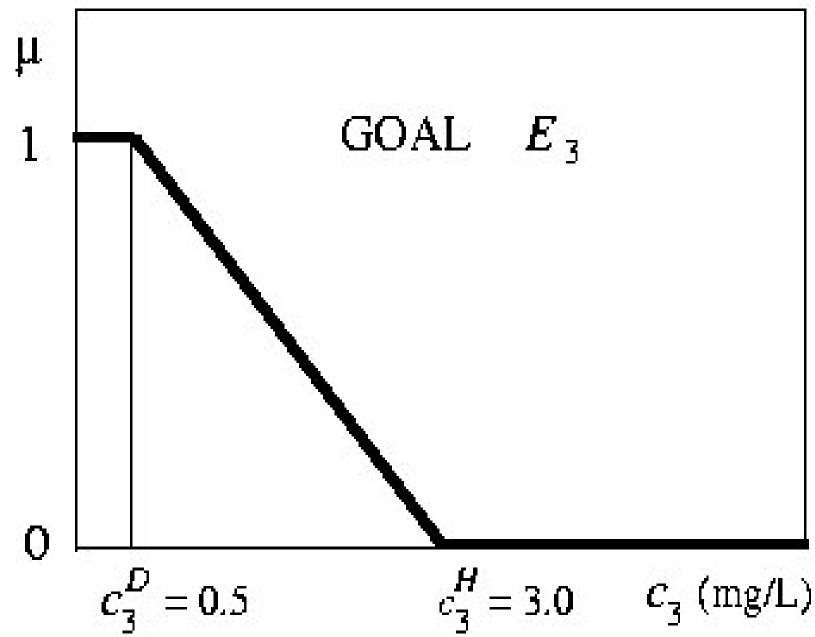
Details of the membership functions for the fuzzy goals

For all Checkpoints l In reach r_e (1)	Goal E_l		Goal F_m		
	C_l^H *(mg/L)* (2)	C_l^D (mg/L) (3)	Discharger (4)	$x^L{}_m$ (5)	$x^M{}_m$ (6)
r_1	3.5	0.0	D_1	0.25	0.75
r_2	3.0	0.5	D_2	0.35	0.80
r_3	3.5	0.0	D_3	0.30	0.85
r_4	3.5	0.5	D_4	0.35	0.75
r_5	3.0	0.0	D_5	0.35	0.80
r_6	3.0	0.5	D_6	0.25	0.90
r_7	4.0	1.0	D_7	0.35	0.90
r_8	3.5	1.5	D_8	0.35	0.85
r_9	4.0	1.5	D_9	0.30	0.75

Example – 2 (Contd.)

Fuzzy Goals of the Pollution Control Agency

- Fuzzy Goal E_l : Make the concentration level, C_l, at the checkpoint l as close as possible to the desirable level, $C^D{}_l$ so that the water quality at the checkpoint l is enhanced at l.

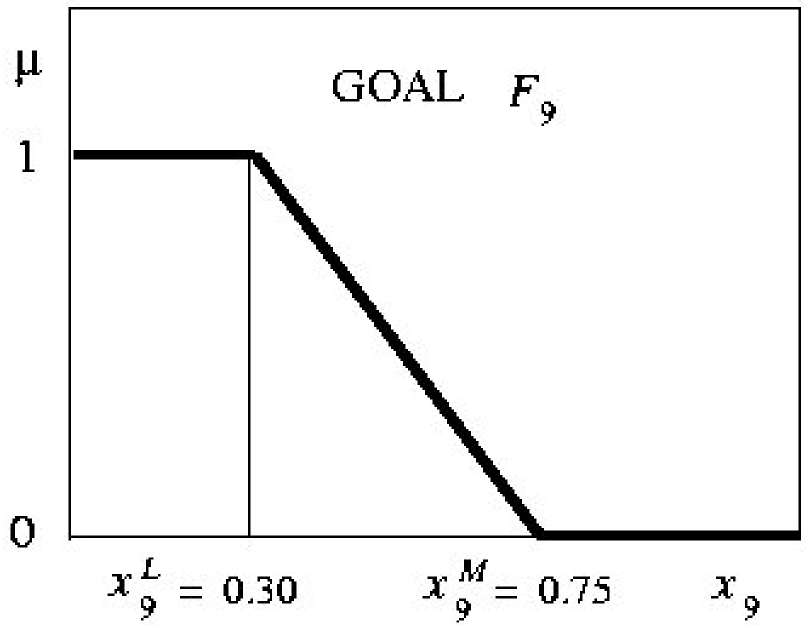


Membership Function for Goal E_3

Example – 2 (Contd.)

Fuzzy Goals of the Dischargers

- Fuzzy Goal F_m : Make the fraction removal level x_m as close as possible to the aspiration level x^L_m for the discharger m.



Membership Function for Goal F_9

Example – 2 (Contd.)

A minimal fraction removal level of 0.30 is imposed by the pollution control agency on all the dischargers . Results are as follows

Discharger	Fraction Removal Level	River Reach R_e	Minimum DO Concentration (mg/L)
(1)	(2)	(3)	(4)
D_1	0.64	r_1	9.89
D_2	0.70	r_2	8.76
D_3	0.72	r_3	8.50
D_4	0.66	r_4	8.80
D_5	0.70	r_5	9.17
D_6	0.75	r_6	7.65
D_7	0.77	r_7	6.90
D_8	0.74	r_8	6.61
D_9	0.49	r_9	6.07

Fuzzy Optimization

Fuzzy sets for reservoir storage and release targets:

- Consider that a reservoir storage volume target, T^S, is to be obtained given a minimum release target T^R, and reservoir capacity K.
- Assume that the known release and unknown storage targets must apply in each of the three seasons in a year.
- The objective will be to find the highest value of the storage target, T^S, that minimizes the sum of squared deviations from actual storage volumes and releases less than the minimization release target.

Source: Loucks, D.P and Eelco Van Beek, Water resources systems planning and management, An introduction to methods, models and applications, UNESCO, 2005. (p.139-140)

Fuzzy Optimization

The optimization model is

Minimize $D = \sum_{t=1}^{3}[(T^s - S_t)^2 + DR_t^{\,2}] - 0.001T^S$

s.t.

$$S_t + Q_t - R_t = S_{t+1} \qquad t = 1, 2, 3$$

$$S_t \leq K \qquad t = 1, 2, 3$$

$$R_t \geq T^R - DR_t \qquad t = 1, 2, 3$$

Assume $K = 20$, $T^R = 25$ and the inflows Q_t are 5, 50 and 20 for time periods $t = 1, 2, 3$.

Source: Loucks, D.P and Eelco Van Beek, Water resources systems planning and management, An introduction to methods, models and applications, UNESCO, 2005. (p.139-140)

Fuzzy Optimization

Solution is

$$D = 184.4$$
$$T^S = 15.6$$
$$S_1 = 19.4$$
$$S_2 = 7.5$$
$$S_3 = 20.0$$
$$R_1 = 14.4$$
$$R_2 = 27.5$$
$$R_3 = 18.1$$

Source: Loucks, D.P and Eelco Van Beek, Water resources systems planning and management, An introduction to methods, models and applications, UNESCO, 2005. (p.139-140)

Fuzzy Optimization

- If the OF is changed to one of maximizing the minimum membership function value, then the new formulation is

$$\text{Maximize } \mu_{min} = \text{maximize minimum } \{\mu_{St}, \mu_{Rt}\}$$

$$\text{Max } \lambda \quad \text{s.t.} \quad \mu_{St} \ge \lambda \;\; \forall t, \quad \mu_{Rt} \ge \lambda \;\; \forall t$$

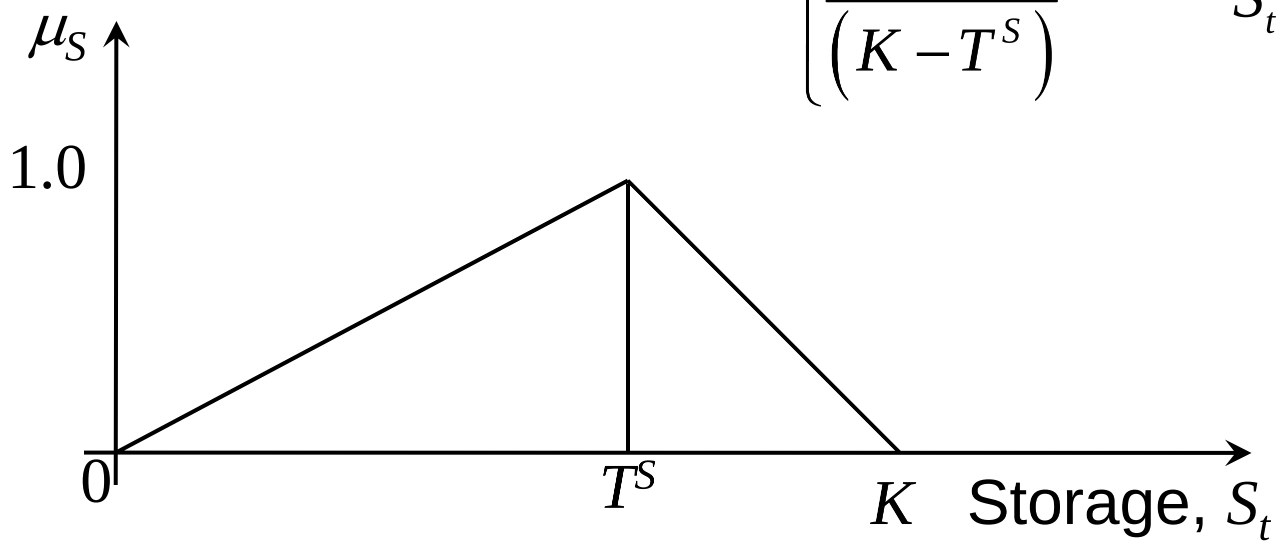
- A common lower bound is set on each membership function, μ_{St} and μ_{Rt}, and this variable is maximized.

Source: Loucks, D.P and Eelco Van Beek, Water resources systems planning and management, An introduction to methods, models and applications, UNESCO, 2005. (p.139-140)

Fuzzy Optimization

- The variables μ_{St} are the degrees of satisfying storage volume target in the three periods t is

$$\mu_{St} = \begin{cases} \dfrac{S_t}{T^S} & S_t \le T^S \\ \dfrac{\left(K - S_t\right)}{\left(K - T^S\right)} & S_t \ge T^S \end{cases}$$

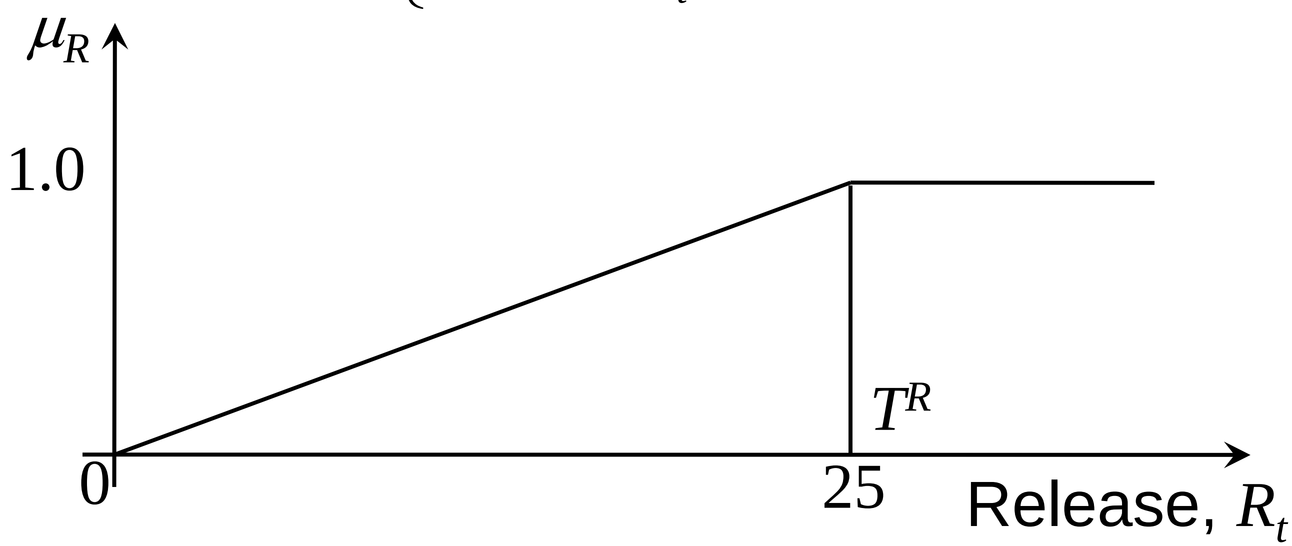


Source: Loucks, D.P and Eelco Van Beek, Water resources systems planning and management, An introduction to methods, models and applications, UNESCO, 2005. (p.139-140)

Fuzzy Optimization

- The variables μ_{Rt} are the degrees of satisfying storage volume target in the three periods t is

$$\mu_{Rt} = \begin{cases} \dfrac{R_t}{T^R} & R_t \leq T^R \\ 1 & R_t \geq T^R \end{cases}$$



Source: Loucks, D.P and Eelco Van Beek, Water resources systems planning and management, An introduction to methods, models and applications, UNESCO, 2005. (p.139-140)

Fuzzy Optimization

- The optimal solution is

$$\mu_{min} = 0.556$$
$$T^S = 20.0$$
$$S_1 = 20.0$$
$$S_2 = 11.1$$
$$S_3 = 20.0$$
$$R_1 = 13.9$$
$$R_2 = 41.1$$
$$R_3 = 20.0$$

Source: Loucks, D.P and Eelco Van Beek, Water resources systems planning and management, An introduction to methods, models and applications, UNESCO, 2005. (p.139-140)