INDIAN INSTITUTE OF SCIENCE

Water Resources Systems:

Modeling Techniques and Analysis

Lecture - 37

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Summary of the previous lecture

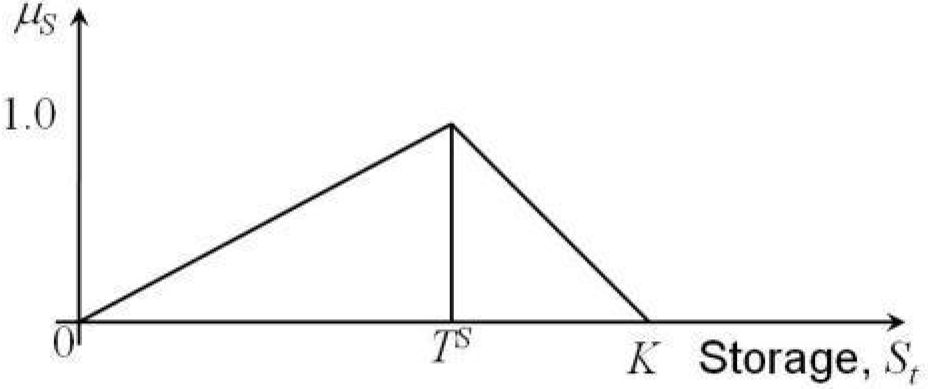
Minimize $D = \sum_{t=1}^{3} \left\{ \left(T^S - S_t \right)^2 + DR_t^2 \right\} - 0.001T^S$

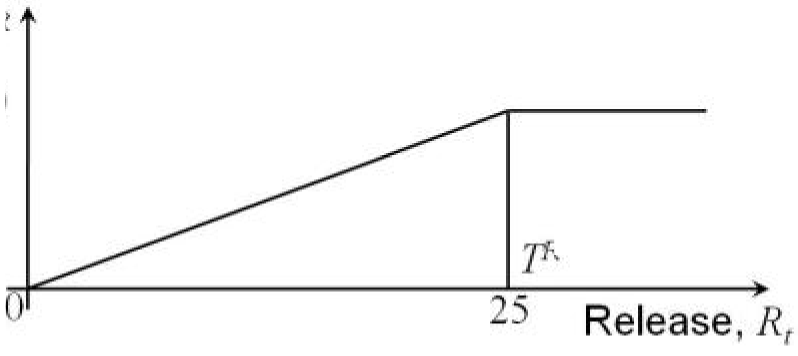
s.t. $S_t + Q_t – R_t = S_{t+1}$ $t = 1, 2, 3$

$S_t \leq K$ $t = 1, 2, 3$

$R_t \geq T^R – DR_t$ $t = 1, 2, 3$

Maximize μ_{min} = maximize minimum $\{\mu_{St}, \mu_{Rt}\}$





CONJUNCTIVE USE OF SURFACE AND GROUND WATER

Conjunctive Use

Conjunctive use of surface and ground water resources:

- Impounding stream water in a surface reservoir – transferred at optimum rate to ground water storage.
- Periods of above normal precipitation
  - Use of surface water
  - Artificial recharge of ground water
- Drought periods
  - Pumping of ground water
  - Lowers ground water levels

Todd, D.K., (1980), Ground Water Hydrology, John Wiley & Sons, Inc.

Conjunctive Use

Surface water and ground water reservoirs:

- SW reservoirs are lost forever once they are silted up. With passage of time suitable sites for new storage reservoirs will be fewer while underground storage spaces will remain practically unaffected by development.
- Yield from ground water sources is more dependable than from surface reservoirs.
- Physical and chemical quality of GW is more uniform that of surface water.

Todd, D.K., (1980), Ground Water Hydrology, John Wiley & Sons, Inc.

Conjunctive Use

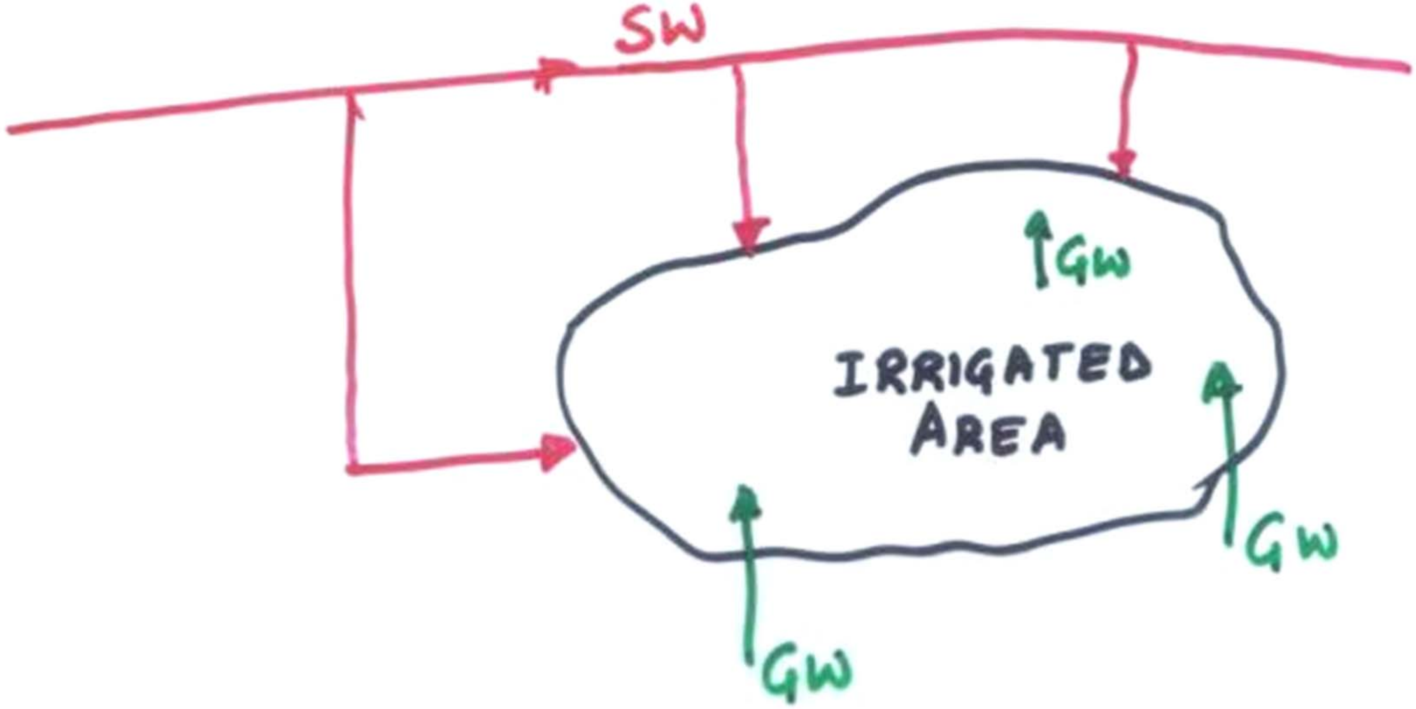
- Ground water development scheme has a short gestation period.
- Saline GW areas need surface water.
- It is practically not possible to divert all surface water to underground even if the operation were profitable.

Todd, D.K., (1980), Ground Water Hydrology, John Wiley & Sons, Inc.

Conjunctive Use

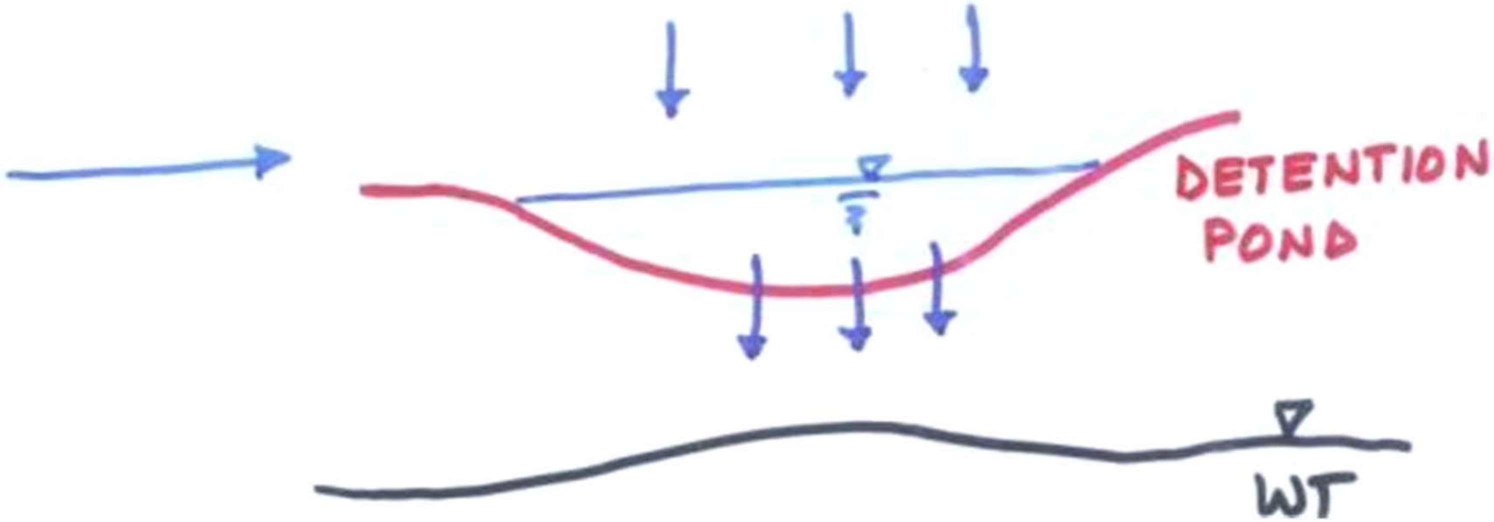
Integrated plan:

- Allocation of surface and ground waters



Conjunctive Use

- Artificially recharging ground water.



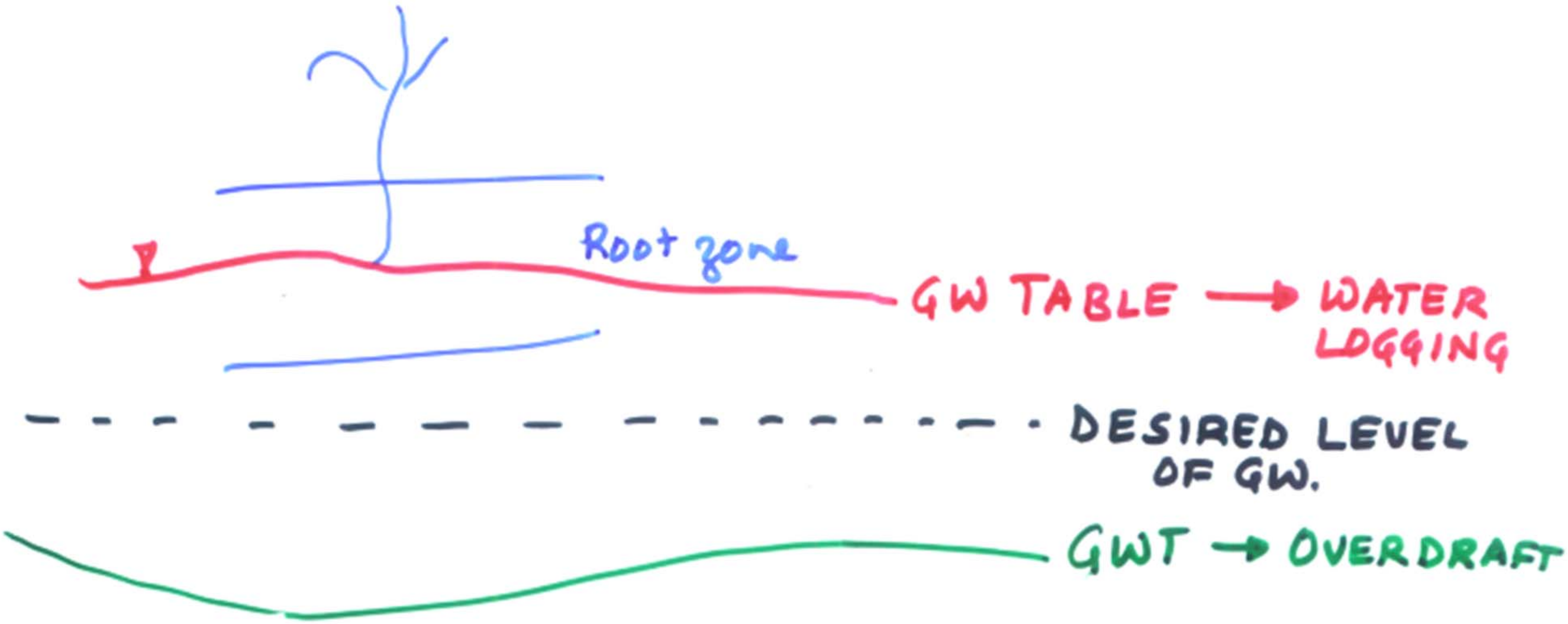
- Preventing undesirable effects – water salination, increased operation costs, overdraft of GW basins.

Conjunctive Use

Objectives :

- Water supply management objective
  - To meet water demands at lowest overall cost
- Water quality objective
  - To maintain concentration levels of water quality constituents.
- Prevention of undesirable effects
  - Overdraft of GW basins
  - Salinity
  - Water logging

Conjunctive Use

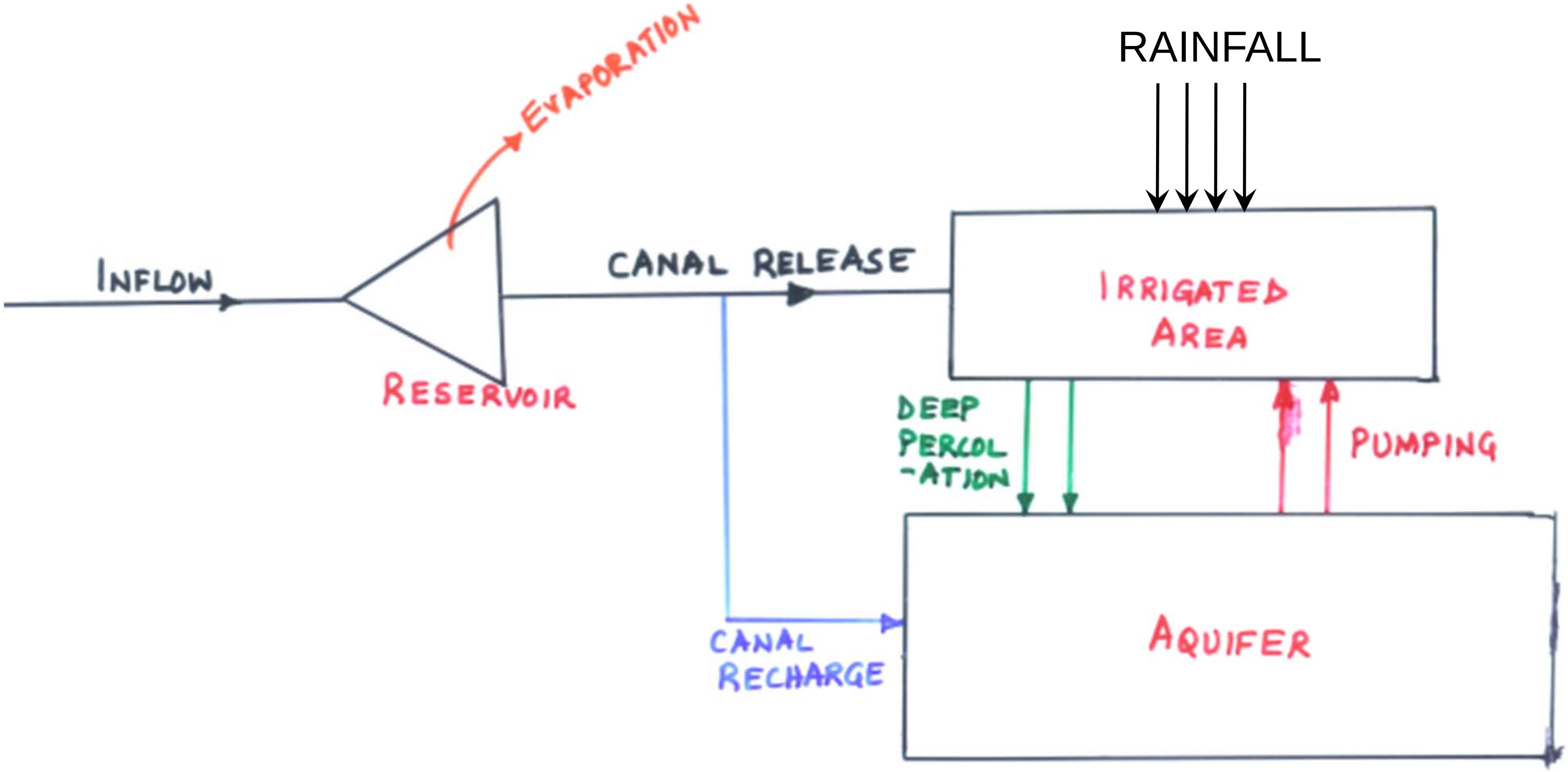


Modeling for Conjunctive Use

Data requirements:

- Surface water resources: Reservoir details, inflow, evaporation, seepage …
- Ground water resources: Aquifer type and boundaries, aquifer parameters, calibrated GW model.
- Geologic conditions
- Water distribution systems
- Water use pattern

Modeling for Conjunctive Use



Modeling for Conjunctive Use

Model formulation:

Monthly Time Periods

- Decision variables:
  - Reservoir release during period t: R_t
  - Ground water pumping during period t: GW_t

    $t = 1, 2, \dots. 12$ (monthly periods)

- Objective:
  - To maximize the net benefits in a year

Modeling for Conjunctive Use

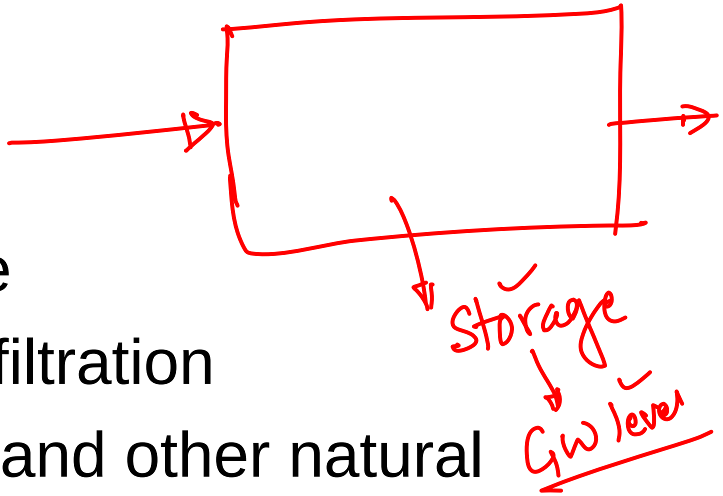
- Constraints:
  - Ground water balance ✓
  - Surface water balance ✓
  - Minimum and maximum drawdown ✓
  - Reservoir storage limits ✓
  - Meeting irrigation requirements ✓
  - Total GW pumping in a year ✓

Modeling for Conjunctive Use

Components of GW balance:

GW inflow – GW outflow = Change in GW storage

$$S_{gw} = W_p + W_r + W_c + W_{as} + W_{ag} + GW_i - GW_b - GW_e - GW_o - GW_{ET} \pm GW_n$$



where

S_{gw}= volume change in GW storage

W_p = recharge from precipitation infiltration

W_r = recharge from streams, lakes and other natural water bodies

Modeling for Conjunctive Use

W_c = recharge by storage structures, canals, distributaries and other irrigation works

GW_i = ground water inflow

W_{as} = recharge from surface water applied for irrigation

W_{ag} = recharge from return circulation of GW applied for irrigation

GW_b = GW discharge to streams and springs

GW_e = GW extraction by pumping and flowing wells

GW_o = GW outflow

GW_{ET} = ET loss of GW from phreatophytic vegetation

GW_n = other items, if any (e.g., artificial recharge through injection well)

Modeling for Conjunctive Use

Recharge components depend on:

- Geology
- Intensity and duration of rainfall
- Evapotranspiration
- Soil moisture
- Runoff
- Infiltration capacity of soil
- Storage characteristics of aquifers
- Movement of GW
- Flow in the unsaturated zone

Modeling for Conjunctive Use

Components of SW balance:

$$S_{t+1} - S_t = I_t - R_t - E_t$$

where

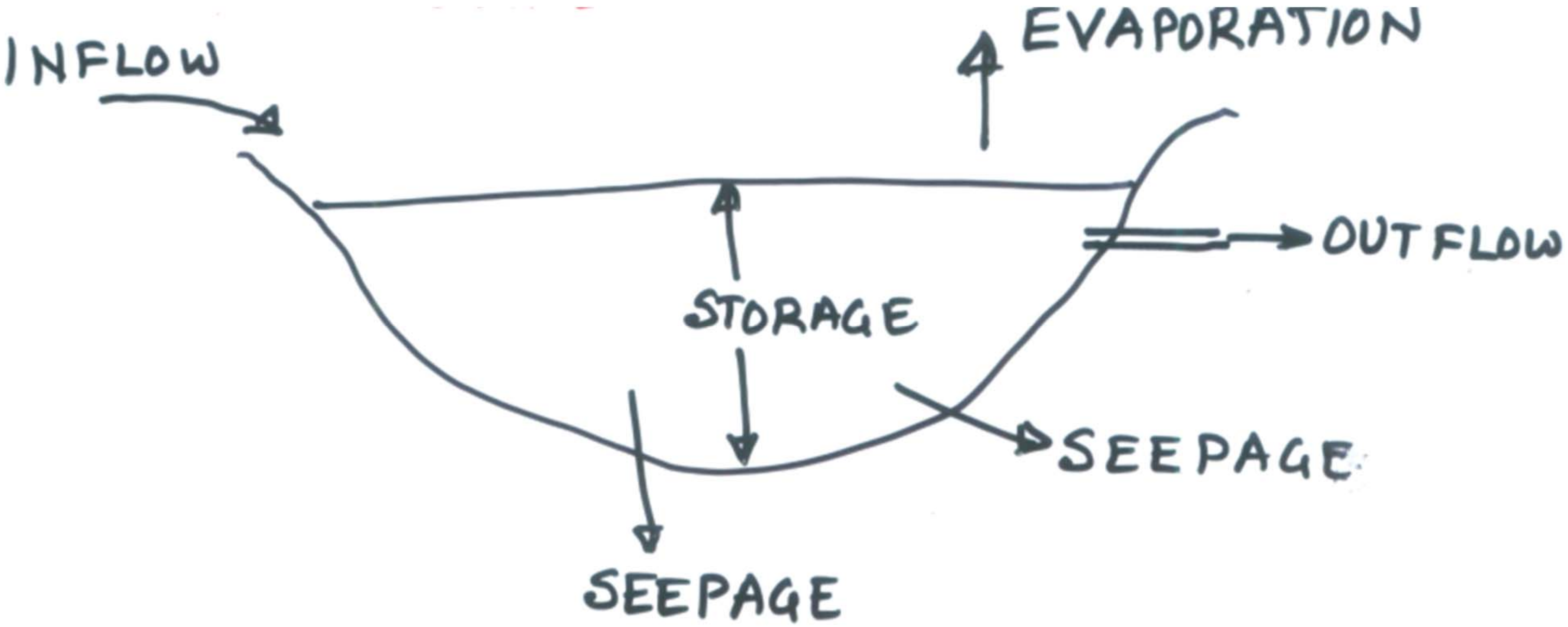
S_t = storage at the beginning of period (e.g., month) t

I_t = inflow during period t

R_t = outflow (release) during period t

E_t = evaporation and other losses during period t

Modeling for Conjunctive Use



Modeling for Conjunctive Use

Example : From drawdown conditions,

- Total GW that can be pumped in a year is Q_p
- Canal recharge: 30% of release
- Recharge from irrigation: 10% of water applied
- Inflow in period t : $\{I_t\}$
- Net benefits for each unit of water applied in period t: B_t
- Ground water storage in period t : GS_t
- $G_{min} < GS_t < G_{max}$
- Reservoir capacity : S_{max}
- Irrigated demand in period t : $\{D_t\}$

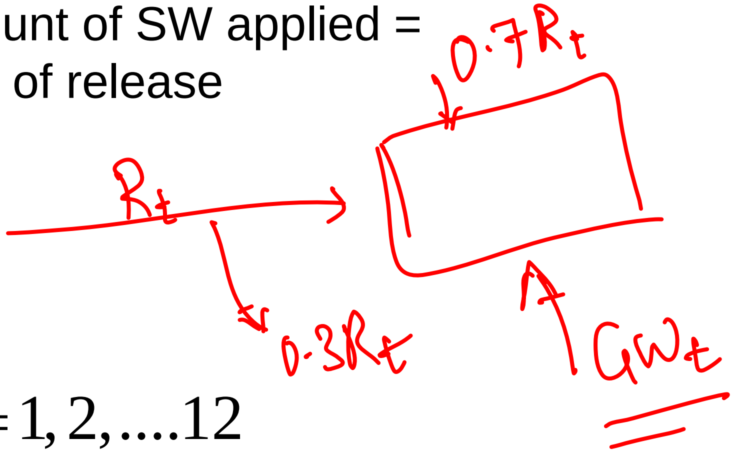
Modeling for Conjunctive Use

Max $$\sum_{t=1}^{12} B_t \left(GW_t + 0.7R_t \right)$$

Benefit per unit water applied at the field

Amount of GW applied

Amount of SW applied = 70% of release



s.t.

- Reservoir storage continuity

$$S_{t+1} = S_t + I_t - R_t - E_t \qquad t = 1, 2, \ldots .12$$

- Ground water balance

$$GS_{t+1} = GS_t + 0.3\left(R_t\right) + 0.1\left(GW_t + 0.7R_t\right) - GW_t$$

pumping

$$t = 1, 2, \ldots .12$$

Recharge due to : Canal seepage

Deep percolation

Modeling for Conjunctive Use

- Minimum and maximum drawdown

$$G_{min} \le \underline{\underline{GS_t}} \le G_{max} \qquad t = 1, 2, ...12$$

- Reservoir storage limits

$$\underline{\underline{S_t}} \le S_{max} \qquad t = 1, 2, ...12$$

- Irrigation requirement

$$\underset{\substack{\nearrow \\ \text{GW application}}}{GW_t} + 0.7\underset{\substack{\uparrow \\ \text{SW application}}}{R_t} \ge \underset{\substack{\nwarrow \\ \text{Demand}}}{D_t} \qquad t = 1, 2, ...12$$

Modeling for Conjunctive Use

- Constraint on total GW pumping

$$\underbrace{\sum_{t=1}^{12} GW_t}_{\text{Total pumping in a year}} \leq Q_p \qquad \cancel{t = 1, 2, ...12}$$

Maximum permissible pumping in a year

- End of the year storage

$$S_{13} = S_1$$

$$GS_{13} = GS_1$$

End of the year storage = beginning of year storage for next year

- Non-negativity

$$\left.\begin{array}{l} R_t \geq 0; \;\; GW_t \geq 0 \\ \\ S_t \geq 0; \;\;\; GS_t \geq 0 \end{array}\right\} t = 1, 2, ...12$$

Modeling for Conjunctive Use

- Data

Period t (month)	Inflow Q_t (Mm^3)	Demand D_t (Mm^3)	Evaporation E_t (Mm^3)
1	70.62	245	10
2	412.75	308	8
3	348.40	308	8
4	142.29	308	8
5	103.78	285	6
6	45.00	190	6
7	19.06	190	5
8	14.27	78	5
9	10.77	65	6
10	8.69	0	8
11	9.48	0	8
12	18.19	0	10

Modeling for Conjunctive Use

- The economic returns per unit of water applied at the irrigation field for the 12 periods are as follows :

B_t: 60, 50, 50, 50, 50, 90, 90, 90, 90, 0, 0, 0.

- Reservoir capacity : 350 Mm^3 ✓
- Volume of water that can be pumped from the aquifer over the year = 1000 Mm^3 ✓
- Maximum volume of ground water that is allowed to be pumped in a period = 200 Mm^3 ✓
- Canal seepage adding to groundwater = 30% ✓ (i.e., all water lost due to seepage adds as recharge to groundwater)
- Recharge due to irrigation applied = 10% ✓

Modeling for Conjunctive Use

- Solution: (All values in Mm^3)

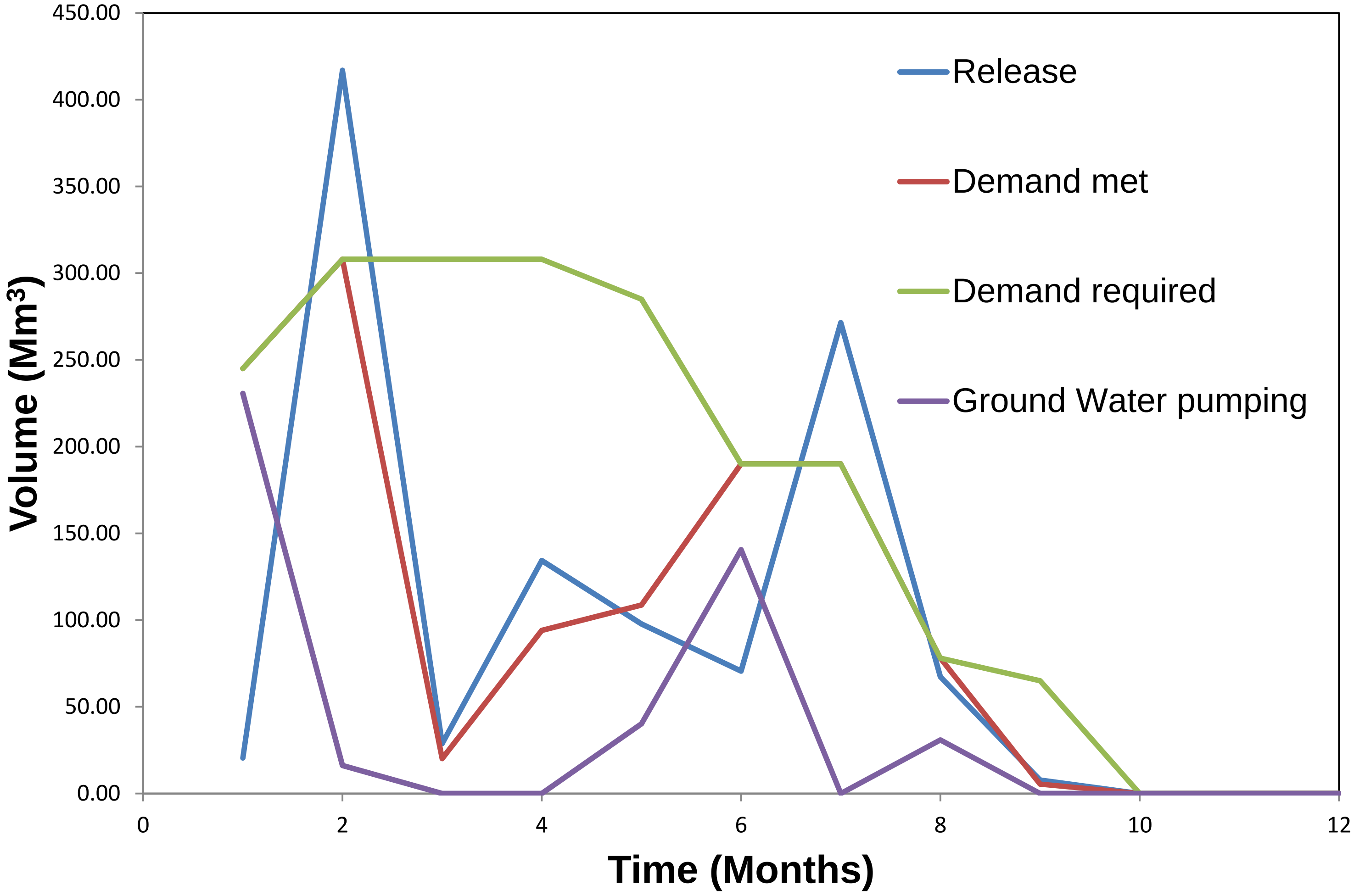
Water applied

Water Demand.

t	GW(t)	R(t)	0.7R(t)+GW(t)	D(t)	RECH(t)
1	230.65	20.50	245.00	245.00	30.65
2	16.19	416.88	308.00	308.00	155.86
3	0.00	28.74	20.12	308.00	10.63
4	0.00	134.29	94.00	308.00	49.69
5	40.20	97.78	108.64	285.00	40.20
6	140.60	70.57	190.00	190.00	40.17
7	0.00	271.43	190.00	190.00	100.43
8	30.87	67.32	78.00	78.00	28.00
9	0.00	7.78	5.44	65.00	2.88
10	0.00	0.00	0.00	0.00	0.00
11	0.00	0.00	0.00	0.00	0.00
12	0.00	0.00	0.00	0.00	0.00

Modeling for Conjunctive Use

$K = 350\ \text{Mm}^3,\ G_{max} = 1000\ \text{Mm}^3$



Conjunctive Use

Ground Water Balance Equation

Two dimensional, unsteady flow in an isotropic, homogeneous, unconfined aquifer is given by (Willis and Yeh, 1992),

$$\frac{\partial}{\partial x}(T\frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(T\frac{\partial h}{\partial y}) = S_y \frac{\partial h}{\partial t} + Q_P - Q_R$$

h	Ground water level (m)
T	Transmissivity m^2/day
S_y	Specific yield
Q_P	pumping rate per unit area m^3/day/m^2
Q_R	Recharge rate per unit area m^3/day/m^2
x and y	Cartesian coordinates in plan
t	time in days