CS 6210: Performance Evaluation of Computer Systems; Aug. 2011, Prof. Krishna Sivalingam
Sample Exercises: Set 1
Please try to solve these by yourself or in groups. You are always welcome to contact the instructors or TAs for clarifications on whether your approach/solution is correct. You will learn better by attempting to solve these, rather than simply going through the solutions.

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1. Assume that the probability of error-free transmission of a message over a communication channel is p. If a message is not transmitted correctly, retransmission is done, until a correct transmission occurs. Assuming that successive transmissions are independent, what is the probability that no retransmission is required? What is the probability that exactly two retransmissions are required? If on average nine transmissions are required, what is the value of p?

2. Assume that the messages arriving to a communication channel in an interval of duration t seconds is Poisson distributed with parameter $0.3t$. Compute the probabilities of the following events:

   (a) Exactly three messages will arrive during a 10 s interval.

   (b) At most 20 messages arrive in a period of 20 s.

   (c) The number of message arrivals in an interval of 5 s is between 3 and 7.

3. Consider independent tosses of a coin that, on each toss, lands on heads (H) with probability p and on tails (T) with probability $q = 1 - p$. What is the expected number of tosses needed for the pattern $HTHT$ to first appear? (Hint: This can be treated as a Markov Chain).

4. Consider a Web server with an average rate of client requests (λ) equal to 0.5 requests per second. Assume that the arrivals are Poisson distributed. What is the probability that an interval of 25 seconds elapses without any requests?

5. Consider two Poisson processes with rates of λ_1 and λ_2 that are arriving to a common shared queue. Prove that the composite arrival process is also Poisson with rate $\lambda_1 + \lambda_2$.

6. Jobs arriving to a compute server have been found to require CPU time that can be modeled by an exponential distribution with parameter $\frac{1}{150}\ ms^(- 1)$. A round-robin scheduler with time quantum of 100 ms is used. A job that does not finish within a given time quantum will be routed back to the tail of the waiting jobs queue (i.e. Ready Queue). Find the probability that an arriving job is forced to wait for the second quantum. Out of 800 jobs arriving in a day, how many are expected to finish within the first time quantum?

7. A student arriving at the bank has to visit two counters for service, in sequence. The service time at the first and second counter are exponentially distributed with respective average service times of 10 and 25 minutes. What is the probability that the student's service time will be greater than 30 minutes?

   If the average service time at both counters is 15 minutes, what is the probability that the student's service time will be greater than 30 minutes?

8. Derive the Laplace transform for the Continuous Uniform Random Variable (a,b).

9. Derive the Probability Generating Function for the Geometric variable with parameter p and obtain $E[X]$ and $E[X^2]$ from the same.

10. The lifetime X (in hours) of a component is modeled by a Weibull distributed with shape parameters $\alpha = 2$. Starting with a large number of components, it is observed that 15% of the components that have lasted 80 hours fail before 90 hours. Determine the scale parameter of the distribution.

11. Determine the channel throughput using the DTMC model of a Slotted Aloha system, given that $m = 3, a = 0.2, b = 0.25$.