CS 6210: Performance Evaluation of Computer Systems; Aug. 2011, Prof. Krishna Sivalingam Sample Exercises: Set 2

Please try to solve these by yourself or in groups. You are always welcome to contact the instructors or TAs for clarifications on whether your approach/solution is correct. You will learn better by attempting to solve these, rather than simply going through the solutions.

Assume Poisson arrival rates and exponential service times unless otherwise indicated.

- 1. Programs make an average of 90 requests to the disk every second. The disk hardware takes an average of 9.7 msec to complete each request. What is the average length of time that a program must wait from when it makes a disk request until it is complete? What is the average number of requests queued? How often are more than 4 requests queued or being served?
- 2. Assume that you have a printer that can print an average file in two minutes. Every two and a half minutes a user sends another file to the printer. How long does it take on average before a user can get their output?

To speed things up you can buy a second printer that is exactly the same as the one you have. How long will it take (on average) for a user to get their files printed if you had two identical printers?

Another alternative is to replace the existing printer with one that can print a file in an average of one minute. How long does it take for a user to get their output with the faster printer?

- 3. Students arrive to meet their instructor in the latter's office, at the rate of λ students per hour. The average time taken to answer each student's questions is 10 minutes. If the average waiting time is required to be less than 30 minutes, what should be the maximum arrival rate per hour?
- 4. A company is trying to determine whether to rent a slow or a fast copier. The company believes that an employees time is worth \$15 per hour. The slow copier rents for \$4 per hour and it takes an employee an average of 10 minutes to complete copying (exponentially distributed). The fast copier rents for \$15 per hour and it takes an employee an average of 6 minutes to complete copying. An average of 4 employees per hour need to use the copying machine (exponential interarrival times). Which machine should the company rent, if it wants to minimize cost?
- 5. Each airline passenger and his or her luggage must be checked to determine whether he or she is carrying a weapon onto the plane. Suppose that at a certain airport, an average of 10 passengers per minute arrive (exponentially distributed). To do the check, the airport must have an X-ray machine and a metal detector. Whenever a checkpoint is in operation, two employees are required. A checkpoint can check an average of 12 passengers per minute (also exponentially distributed). Under the assumption that the airport has only one checkpoint, answer the following questions:

(a) What is the probability that a passenger will have to wait before being checked? (b) On the average, how many passengers are waiting in line to enter the checkpoint? (c) On the average, how long will a passenger spend at the checkpoint?

- 6. Determine the blocking probability for an M/M/5/5 system given $\lambda = 120$ and $\mu = 600$.
- 7. Consider a system in which the birth rate decreases and the death rate increases as the number in the system k increases. That is:

$$\lambda_k = \begin{cases} (K-k)\lambda & \text{if } k \le K \\ 0 & \text{if } k > K \end{cases}$$
$$\mu_k = \begin{cases} k\mu & \text{if } k \le K \\ 0 & \text{if } k > K \end{cases}$$

Write down the equations for p_n , probability of the system being in state n at steady-state.