

CS 6210: Perf. Evaluation of Comp. Systems

Quiz I: Sep. 14, 2011, 2:00 pm - 3:15 pm, Prof. Krishna Sivalingam

Closed Book and Closed Notes; No Cell Phone; ONLY Scientific Calculator; No Sharing of Calculator; No discussions with students enrolled in class.

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Answer the following **SEVEN** questions. Include all calculations and mathematical expression derivations in your answer sheets, for complete credit. Please write legibly. **If you use additional sheets, please write your name and roll number on the additional sheets.**

If you do not present your IIT Madras ID during the Quiz, you have to present it to me within ONE hour of the Quiz; else, the Quiz will not be considered for grading.

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- (1) You have been asked to stand in front of a popular college and count the arriving cars that drop the college students. After measuring for over a week, you find that the average number of cars arriving per hour equals the variance of the number of cars per hour. What random variable distribution will accurately model the number of cars arriving per hour?
- (8) Consider a set of repeated tosses with a coin, where the probability of Heads is  $p$ ,  $0 < p < 1$ . Let random variable  $X$  denote the number of Tails before the  $m^{\text{th}}$  Heads output. Determine  $E[X]$ .
- (8) Derive the Laplace transform of the Exponential random variable. Using this, determine the mean and variance.
- (6) Uncle Shak asked his nephew Dussash to mint a biased coin, who minted a coin with the following property:  
If it lands on **Heads** on a given toss, it will land on **Heads** on the next toss with probability  $a$  and on **Tails** with probability  $(1 - a)$ , where  $0 < a < 1$ .  
If it lands on **Tails** on a given toss, it will land on **Tails** on the next toss with probability  $a$ , and on **Heads** with probability  $(1 - a)$ .  
*Prove or disprove:* In steady state achieved after several repeated tosses, the coin will behave like an unbiased coin, regardless of the initial state.
- (10) Consider a computer system with one CPU and 2 devices. A process runs on the CPU for one time unit and then requests one of the two I/O devices with probability of 0.25 and 0.35 respectively. When the process finishes execution in the current time unit on the CPU, another waiting process is run on the CPU. Each process spends an average of 10 time units being serviced by device 1 and an average of 5 time units being serviced by device 2. **Model the system as a Discrete-time Markov Chain.** What is the average utilization of each disk, under steady-state conditions?
- (8) Show the Markov Chain model for an M/M/1 queue with arrival rate of  $\lambda$ , service rate of  $\mu$ . Assume that  $\lambda < \mu$ . Derive the expression for  $Var[n]$ .
- (9) Consider a single-server queuing system with *discouraged* arrival rates: the Poisson arrival rate when there are  $n$  customers in the system is  $\frac{\lambda}{(n+1)}$ . Let the service time be exponential with parameter  $\mu$ . Derive the expression for  $E[n]$ , i.e. average number of customers in the system, and  $E[r]$ . Is the system always stable?  
If  $E[n] = 2$  and  $\mu = 1$ , what is the value of  $\lambda$ ?