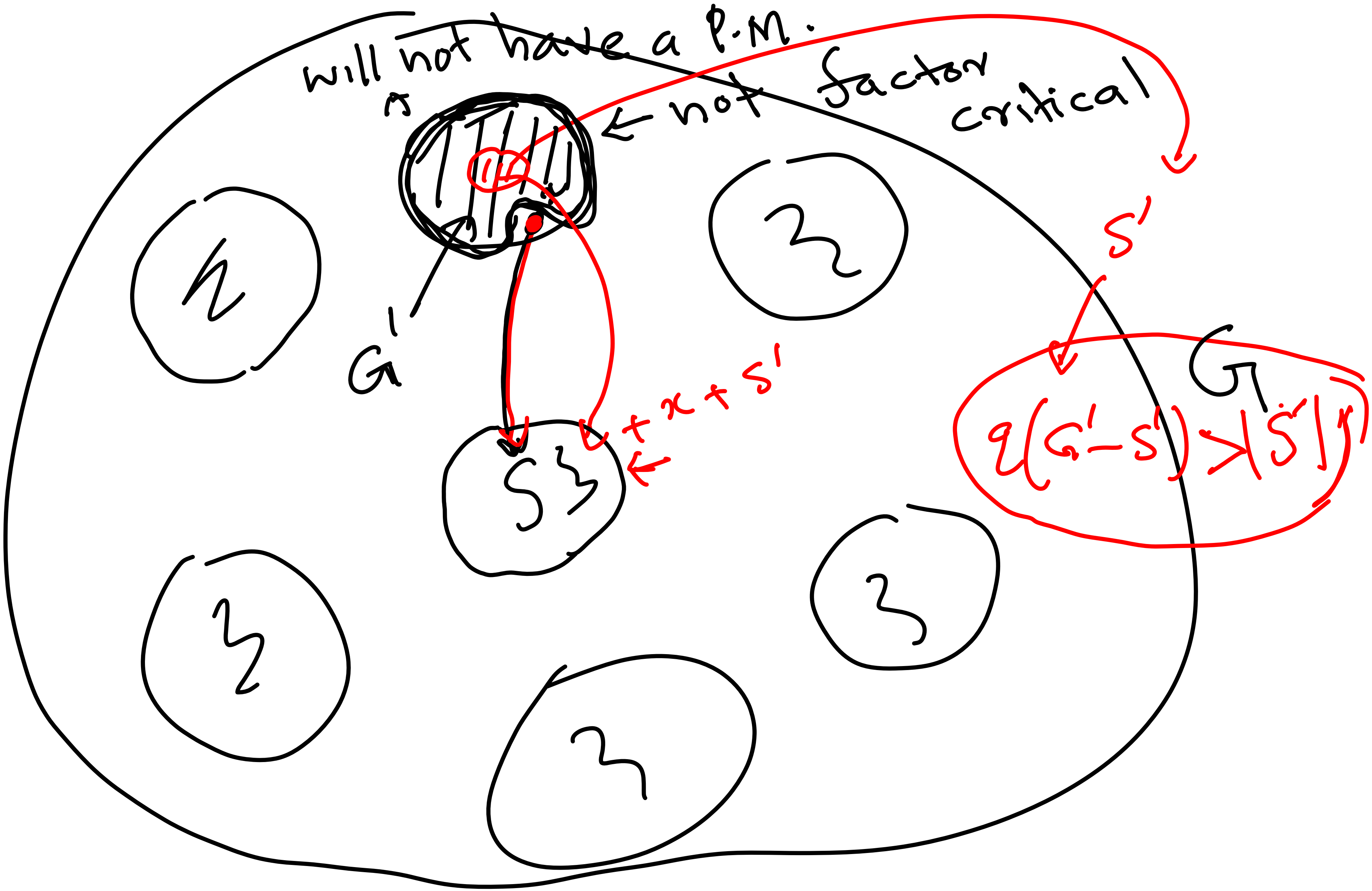
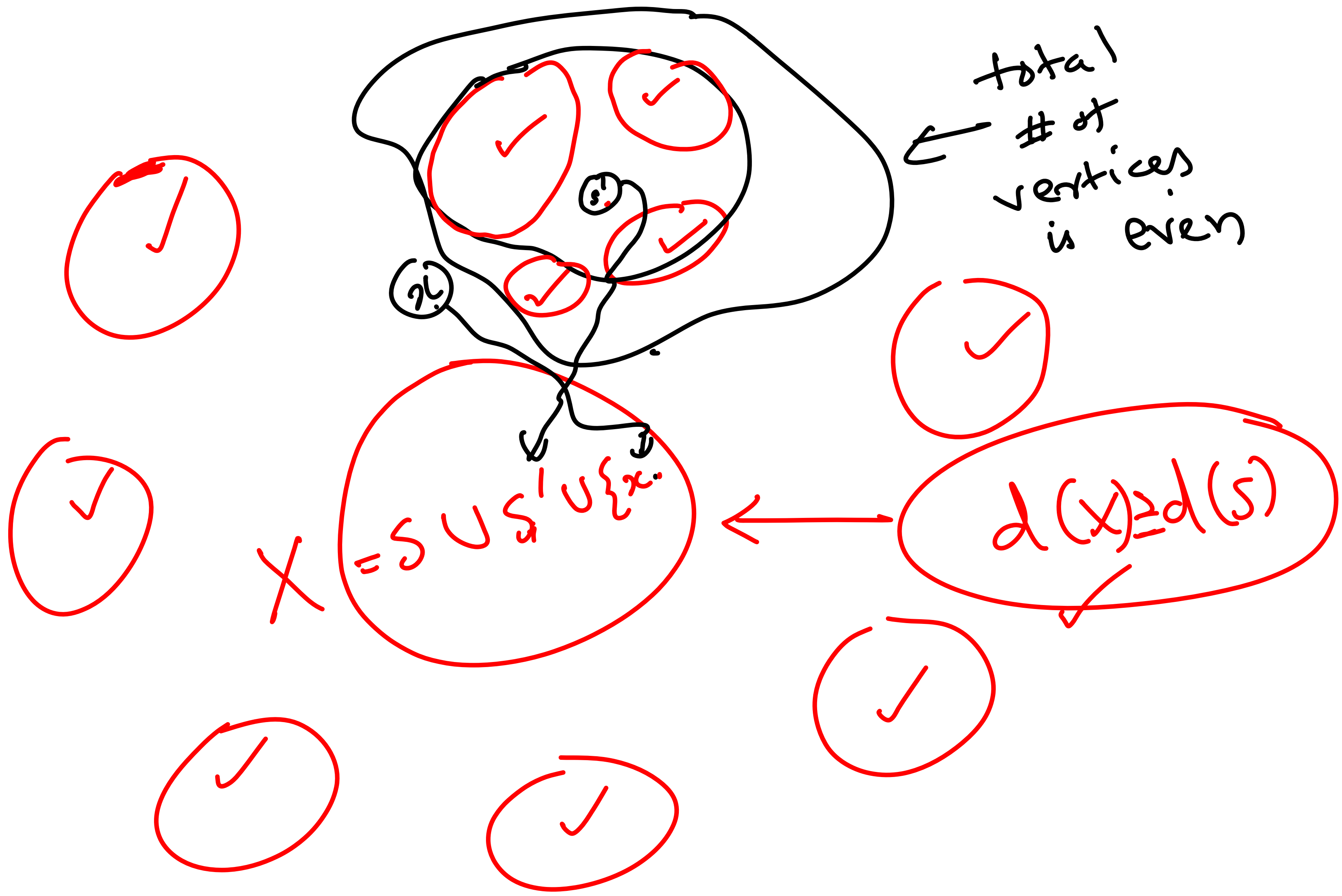
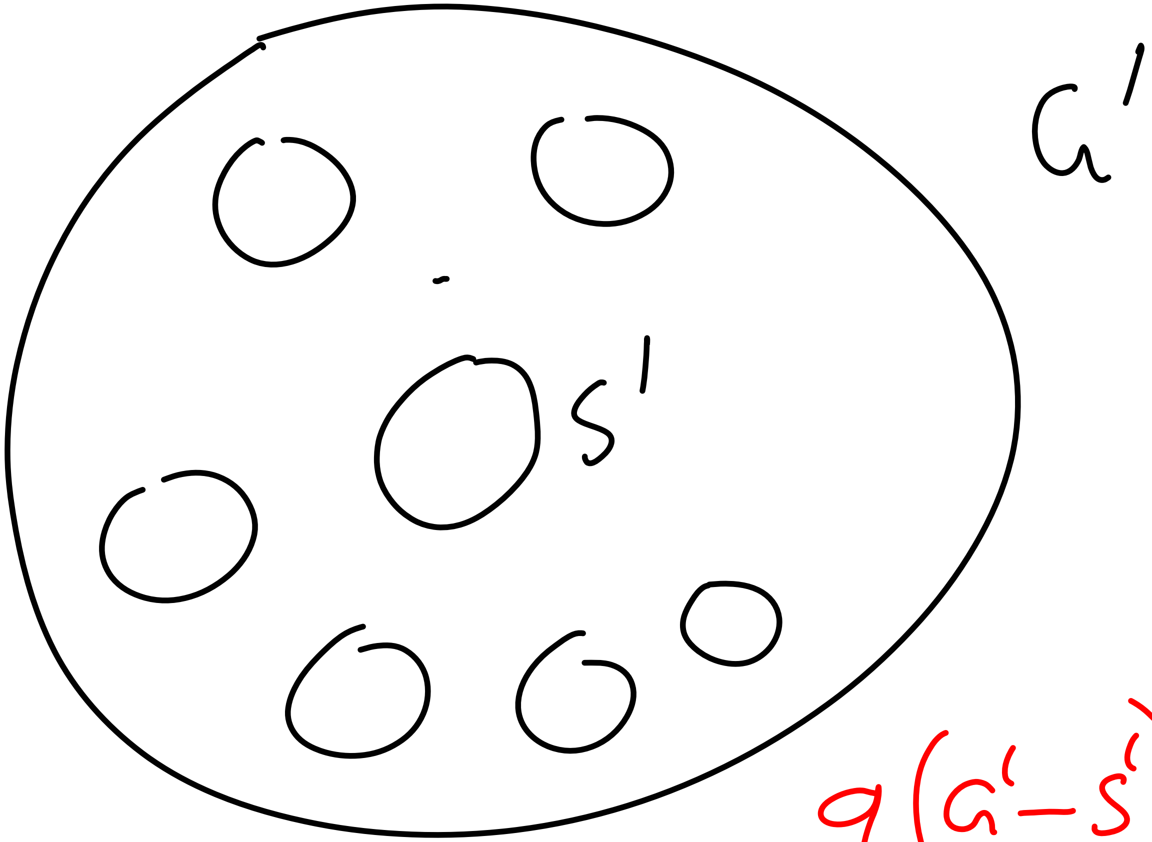


will not have a P.M.
not factor critical
G'
S'
S
$+x+s'$
G
$q(G'-S') > |S'|$


total # of vertices is even
s'
x
X = S ∪ S' ∪ {x}
d(X) ≥ d(S)



$q(G' - S') > |S'|$

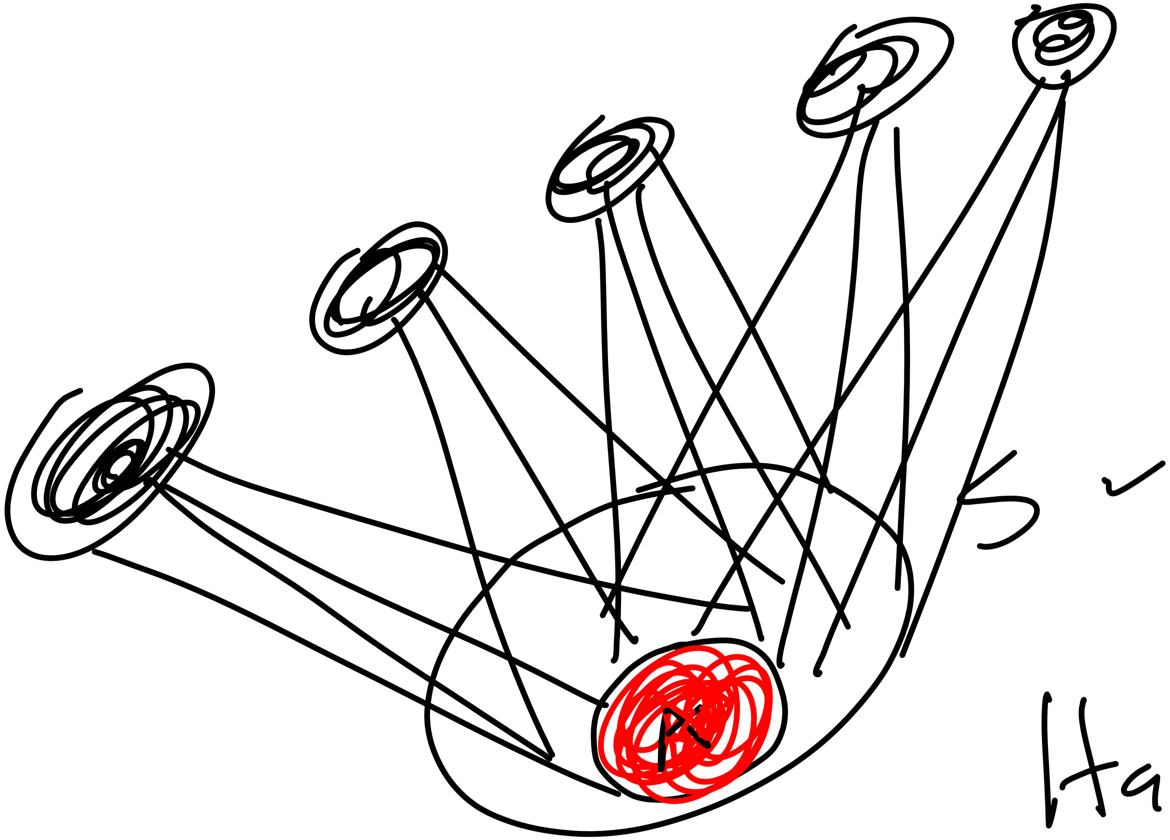
$$q(G'-s') = |s'| + 1$$

$$q(G'-s') \geq |s'| + 2$$

$$d(X) = q(G-X) - |X|$$

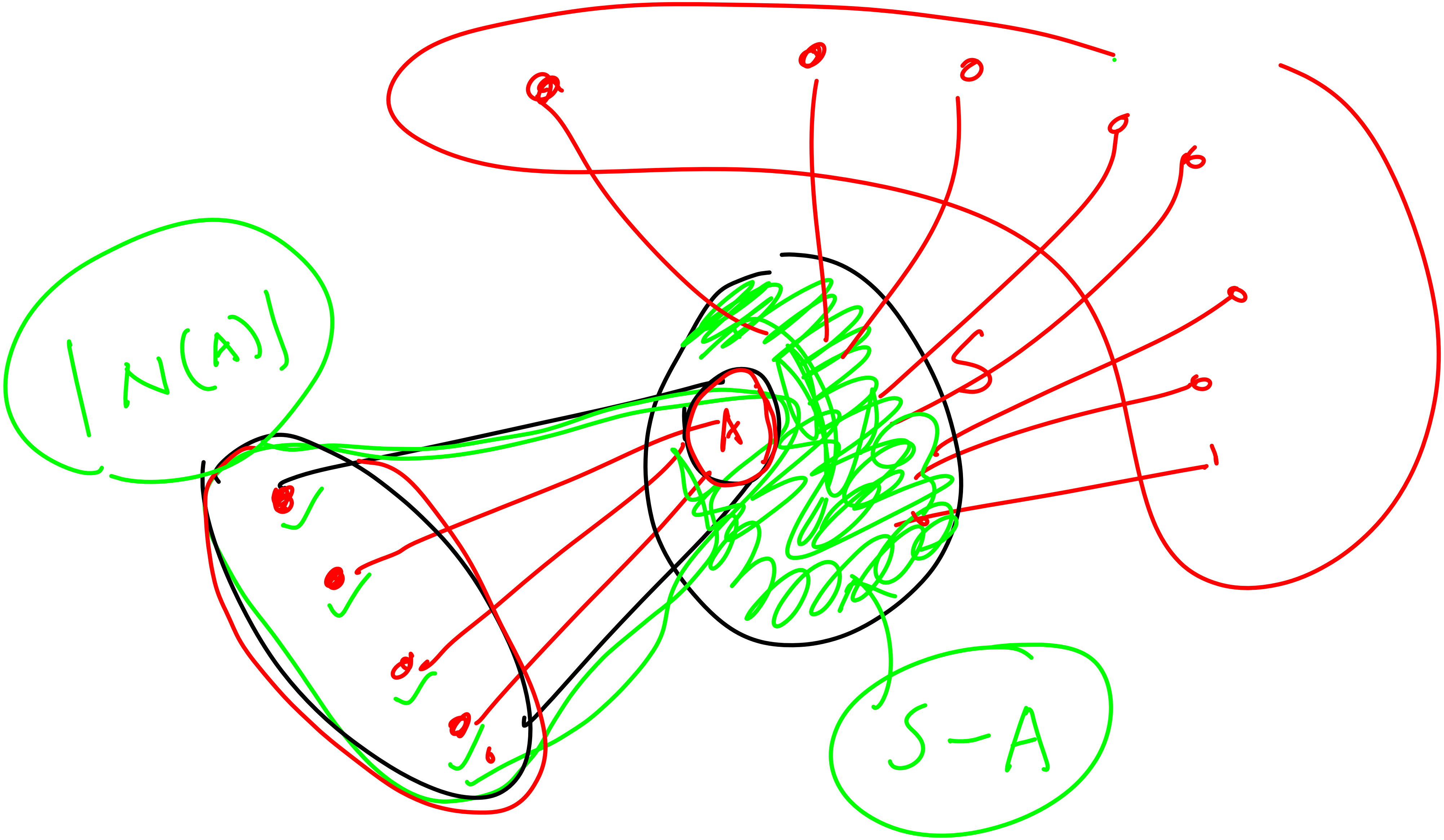
$$\geq \left[q(G-S) - \cancel{1} + \underbrace{\cancel{|S'|} + 2} \right] - \left(|S| + \cancel{|S'|} + \cancel{1} \right)$$

$$X \in \mathcal{F} \qquad \geq q(G-S) - |S| = d(S)$$



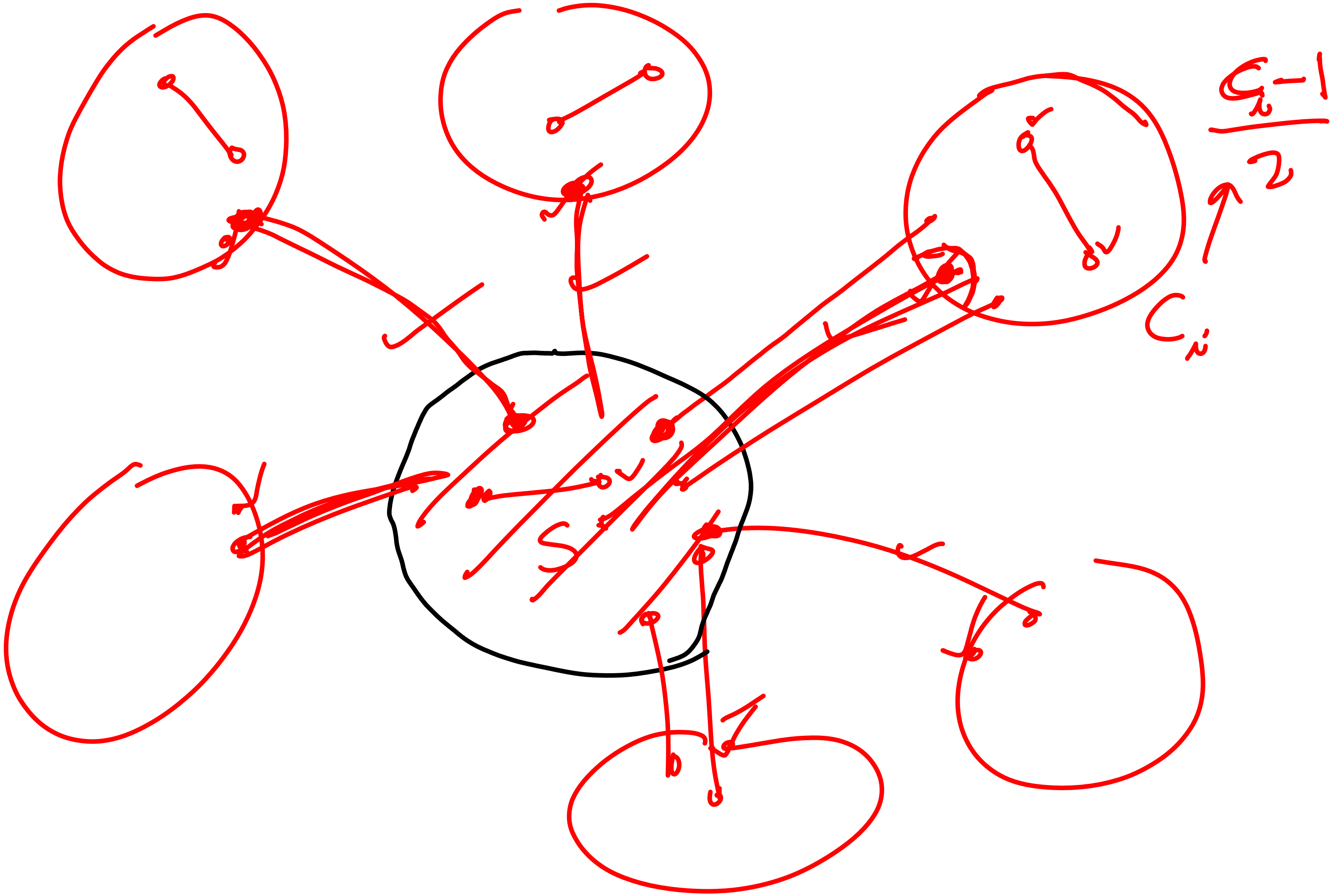
Hall's theorem

$A \subseteq S \quad |N(A)| \geq |A|$



|N(A)|
A
S
S-A

$$
\begin{aligned}
d(S-A) &= d(S|A) \\
&= \underbrace{q(G-S|A) - |S|A|} \\
&> q(G-S) - |S| \\
&= \underline{\underline{d(S)}}
\end{aligned}
$$

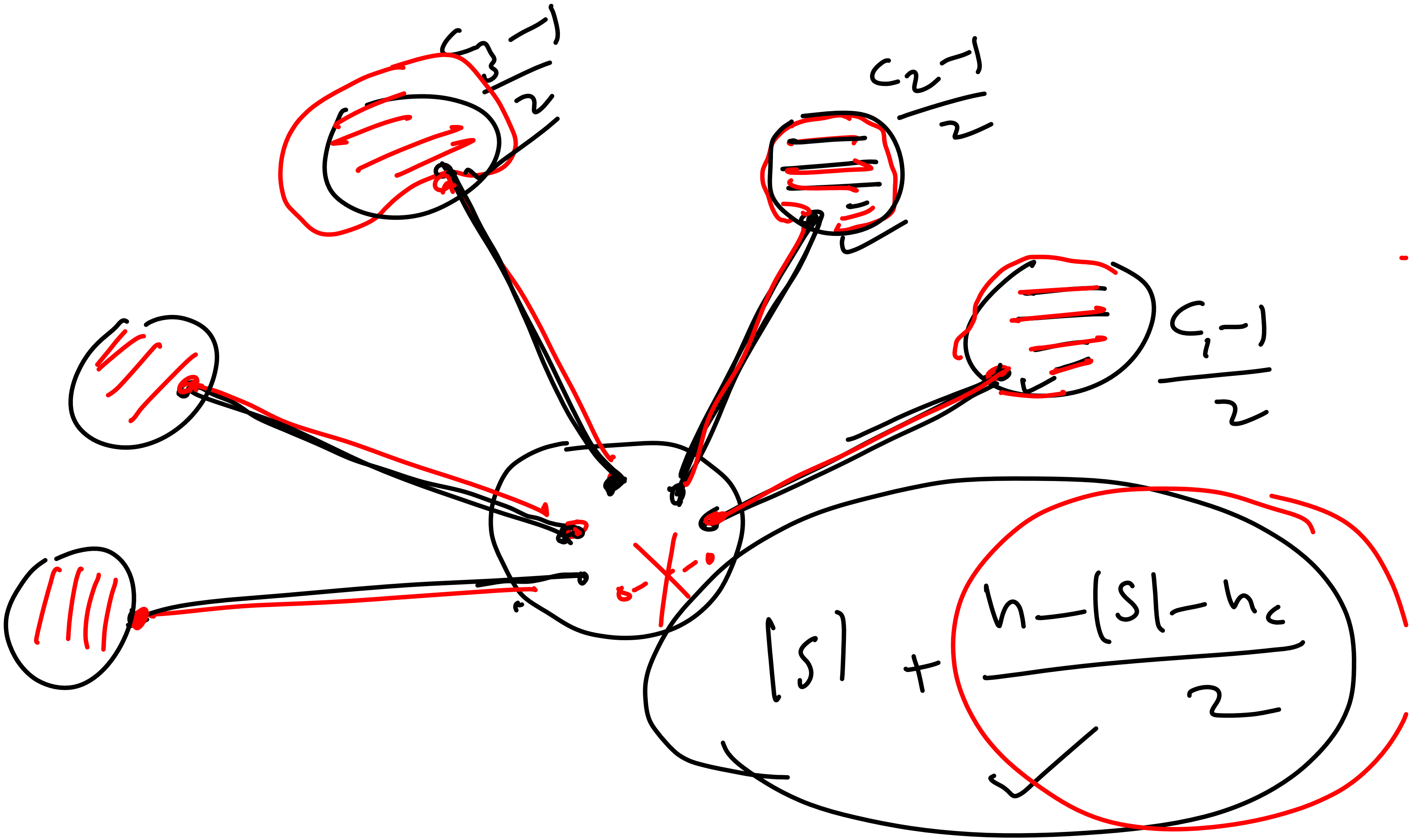

S
C_i
$\frac{c_i - 1}{2}$

M

$$k_c \le \frac{1}{2}(n - n_c - |S|)$$

$$k_s \le |S|$$

$$|M| \le k_c + k_s = \underbrace{|S| + \frac{1}{2}(n - n_c - |S|)}$$

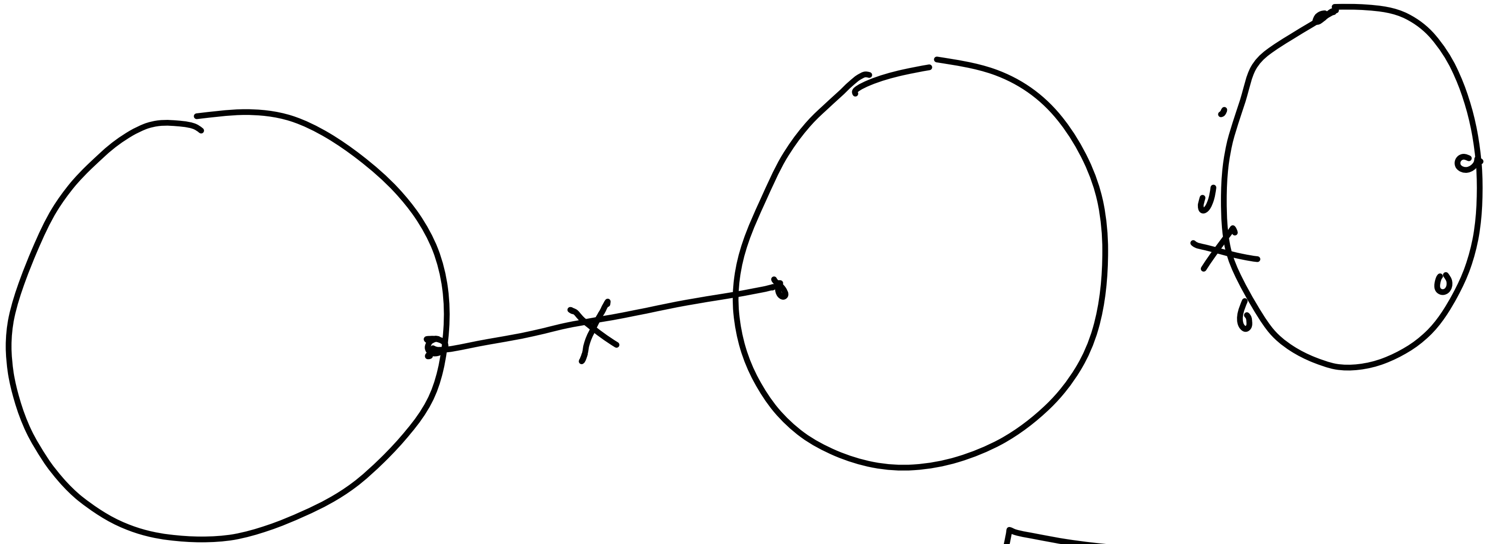


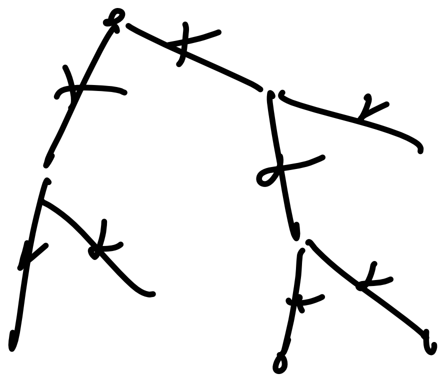
$\frac{c_3-1}{2}$
$\frac{c_2-1}{2}$
$\frac{c_1-1}{2}$
$|S| + \frac{n-(|S|-n_c)}{2}$

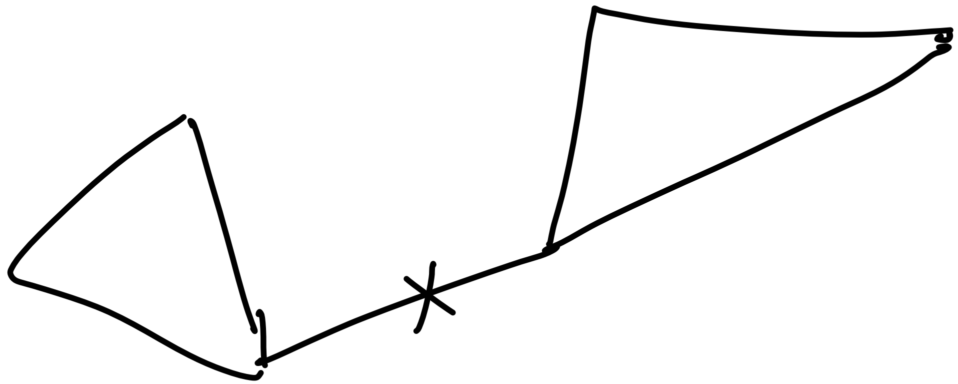
$$k_s = |s|$$

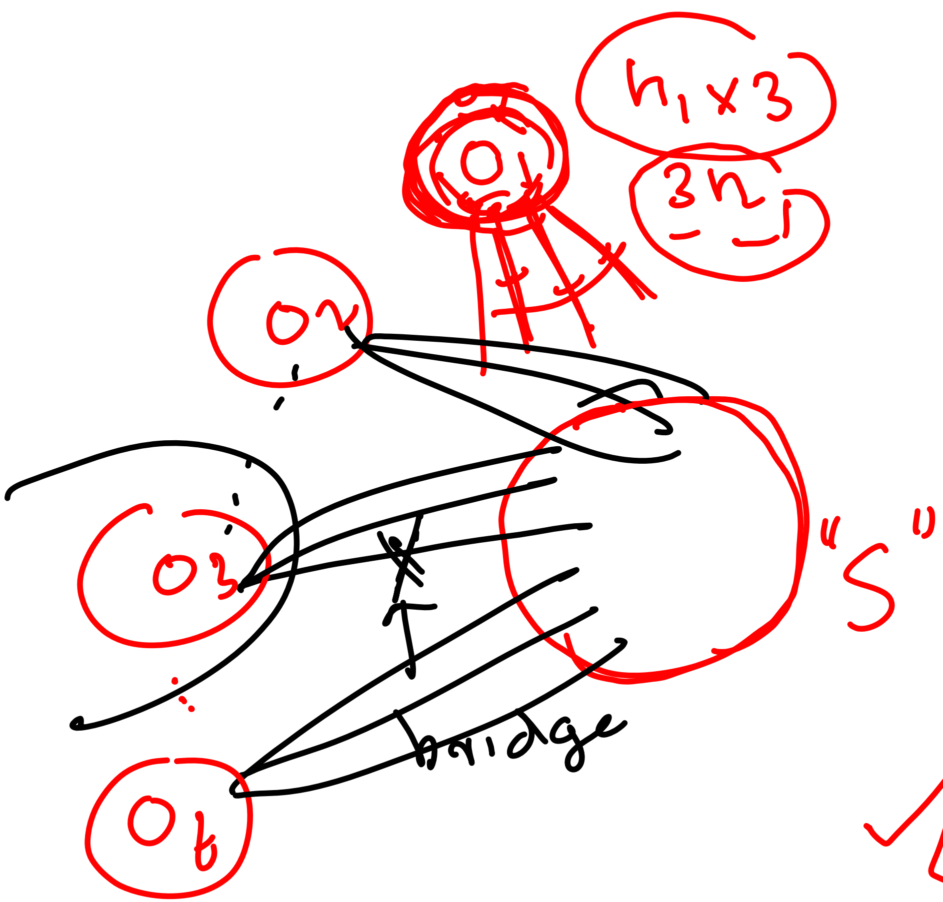
and

$$k_c = \frac{1}{2}\left(n - n_c - |s|\right)$$









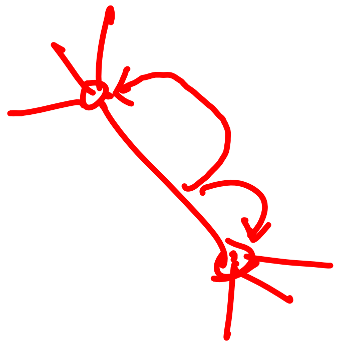
G

(1) bridgeless

(2) cubic

$$\boxed{q(G-S) \leq |S|}$$ ✓

$\sum$ of degree $= 2$ # edges



$$2 \times \#\text{edges} = \sum_{v \in V} \text{of degrees}$$

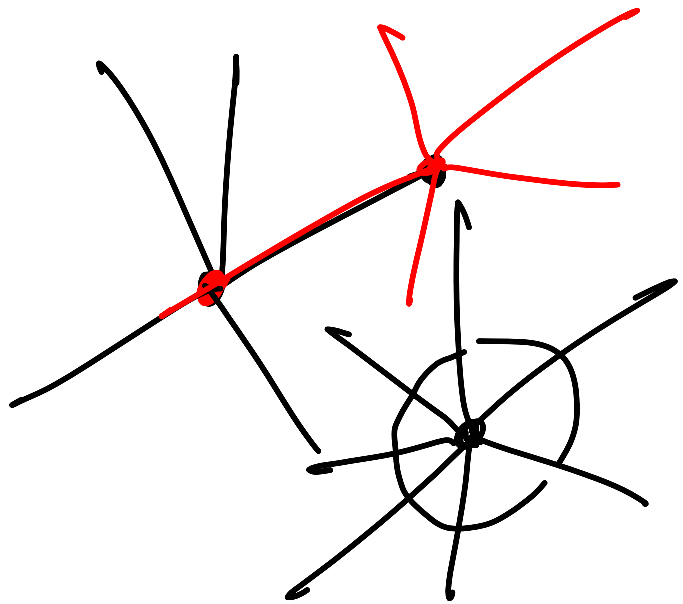
$$q(G-S) \times \cancel{3} \le |S| \times \cancel{3}$$

$$q(G-S) \le |S|$$

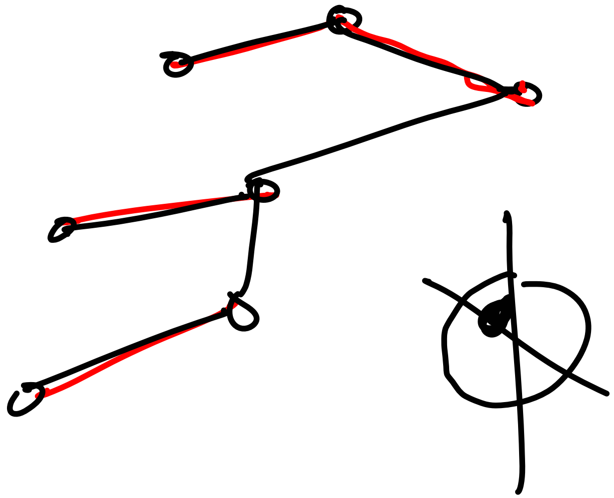
$\alpha(G)$ — independence number

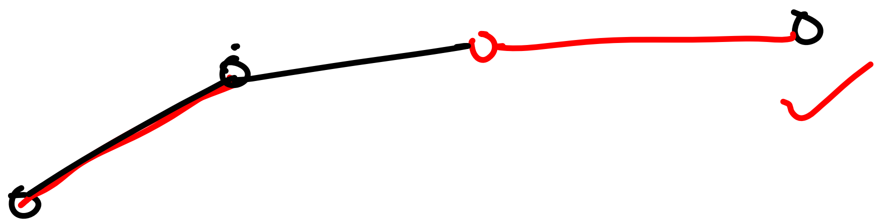
$\alpha'(G)$ —

$\beta(G)$ —



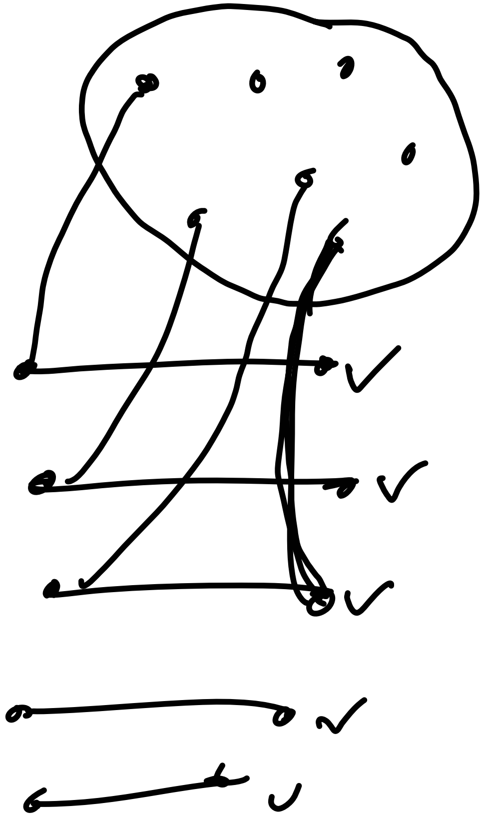
Edge cover





$$\beta(G) + \alpha(G) = n$$

$$\beta'(G) + \alpha'(G) = n$$



$$\alpha'(G)$$

$$\alpha'(G) + n - 2\alpha'(G)$$

$$= n - \alpha'(G) \geq \beta'(G)$$