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References

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What is a Graph?

It is a triple consisting of a vertex set V(G), an edge set E(G) and a relation that associates with each edge two vertices (not necessarily distinct) called its end points.

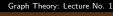
- A loop
- Multiple edge.
- Simple Graph
- Finite Graph

Some simple graphs

- Complete Graph.
- Cycle
- Path

Subgraph and Induced Subgraph

- H is a subgraph of G: Then $V(H) \subseteq V(G)$, $E(H) \subseteq E(G)$. The assignment of end points to edges in H is the same as that in G.
- H is an induced subgraph of G on S, where $S \subseteq V(G)$: Then V(H) = S and E(H) is the set of edges of G such that both the end points belong to S.



A graph ${\it G}$ is connected if each pair of vertices belongs to a path.

Isomorphism

An Isomorphism from a simple graph G to a simple graph H is a bijection $f:V(G)\to V(H)$ such that $(u,v)\in E(G)$ if and only if $(f(u),f(v))\in E(H)$

Forest and Tree

- A graph without any cycle is acyclic.
- A forest is an acyclic graph
- A tree is a connected acyclic graph.

Bipartite Graph

A graph G is bipartite if V(G) is the union of two disjoint (possibly empty) independent sets called partite sets of G. (A subset $S \subseteq V(G)$ is an independent set if the induced subgraph on S contains no edges.)

- A tree is a bipartite graph.
- Can we say that the complete graph K_n is bipartite?
- The complete bipartite graph.

Vertex Cover

A set $S \subseteq V(G)$ is a vertex cover of G (of the edges of G) if every edge of G is incident with a vertex in S.

A vertex cover of the minimum cardinality is called a minimum vertex cover. We will denote this set by MVC(G).

- What is the cardinality of MVC in the complete graph K_n ?
- What about the complete bipartite graph $K_{m,n}$?
- The cycle C_n , when n is even and odd?

The cardinality of a biggest independent set in G is called the independence number (or stability number) of G and is denoted by $\alpha(G)$.

Is there any relation between |MVC(G)| and $\alpha(G)$?

- If we remove a VC from G, the rest is an independent set.
- So, if we remove MVC from G, the rest, i.e. V MVC is an independent set.
- So, $\alpha(G) \ge n |MVC(G)|$. Thus $|MVC(G)| \ge n \alpha(G)$.
- Similiarly if we remove any independent set from G, the rest is VC, and so $|MVC| \le n \alpha(G)$.
- Thus we get $|MVC| = n \alpha(G)$.
- If we denote —MVC(G)— by $\beta(G)$, then we have $\beta(G) + \alpha(G) = n$.

Matching

- A set *M* of independent edges in a graph is called a matching.
- M is a matching of $U \subseteq V(G)$, if every vertex in U is incident with an edge in M.
- Then a vertex in *U* is a matched vertex. The vertices which are not incident with any edge of *M* is unmatched.

A matching M is a perfect matching of G, if every vertex in G is matched by M.

If G has n vertices, what is the cardinality of a perfect matching M of G?

The cardinality of the biggest matching in G can be denoted by $\alpha'(G)$.

What is the value of $\alpha'(G)$ for:

- Cycle *C_n*
- \blacksquare Path P_n
- \blacksquare Complete Graph K_n
- Complete Bipartite graph $K_{m,n}$