Graph Theory: Lecture No. 6

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For any graph G, there exists $S \subseteq V(G)$ such that the following two properties are satisfied:

1. **Consider the bipartite graph obtained by contracting each component of $G - S$ and deleting the edges with both end points in S. In this graph, there is a matching of S.**
2. **The induced subgraph on each component is factor critical.**

Let $d(A) = q(G - A) - |A|$**, for a subset** $A \subseteq V(G)$**.**
Let $\mathcal{F} = \{A : d(A)$ **is maximum** $\}$**.**
Let $S \in \mathcal{F}$ **be such that** $|S| = \max_{A \in \mathcal{F}} |A|$
We will show that such as set S **has the desired two properties.**

First step: We show that each component of $G - S$ is odd.
Second step: We show that each component of $G - S$ is factor critical.
Third step: We show the first property- i.e. S is matchable in the bipartite graph obtained by contracting each component of $G - S$ to a single vertex etc.

The existence of such a set allows us to infer something more about the structure of maximum matchings in a graph G.

Every bridge-less cubic graph has a perfect matching.

For any S, each odd component has at least 3 edges going to S. Thus there are at least $q(G-S).3$ edges reaching S. Since the degree of each vertex is only 3, we get $|S|.3 \geq q(G-S).3$ and therefore Tutte's condition is satisfied.

We studied independent set, matching, vertex cover.
If there is no isolated vertices in the graph G, we can talk of the edge cover of G. Minimum cardinality of an edge cover is denoted by $\beta'(G)$.

$$\beta'(G) + \alpha'(G) = n.$$