Graph Theory: Lecture No. 10

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If G **is** 3**-connected and** $|G| > 4$ **then** G **has an edge** e **such that** G/e **is again** 3**-connected.**

A graph G is 3-connected if and only if there exists a sequence $G_0, G_1, \ldots, G_n$ of graphs such that the following properties hold.
(1) $G_0 = K_4$ **and** $G_n = G$**.**
(2) G_{i+1} **has an edge** (x, y) **with** $d(x), d(y) \geq 3$ **and** $G_i = G_{i+1}/(x, y)$**, for every** $i < n$**.**

Given sets $A, B \subset V(G)$ we call a path $P = (x_0, \ldots, x_k)$ an $A - B$ path if $A \cap V(P) = \{x_0\}$ and $B \cap V(P) = \{x_k\}$

If $A, B \subseteq V(G)$ and $X \subseteq V(G)$ is such that every $A - B$ path in G contains a vertex in X, then we say the X separates the sets A and B in G.

Let $G = (V, E)$ be a graph and $A, B \subseteq V$. Then the minimum number of vertices separating A from B in G is equal to the maximum number of disjoint $A - B$ paths in G.

For $B \subseteq V$ and $a \in V - B$, the minimum number of vertices $\neq a$ separating a from B in G is equal to the maximum number of paths forming an $a - B$ fan in G.

Let a and b be two distinct vertices in G. Then if $(a, b) \notin E$, then the minimum number of vertices $\neq a, b$ separating a from b in G is equal to the maximum number of internally vertex disjoint $a - b$ paths in G.

A graph is k-connected if and only if it contains k independent paths between any two vertices.