Graph Theory: Lecture No. 11

L. Sunil Chandran

Computer Science and Automation,
Indian Institute of Science, Bangalore
Email: sunil@csa.iisc.ernet.in

Let $G = (V, E)$ be a graph and $A, B \subseteq V(G)$. Then the minimum number of vertices separating A from B in G is equal to the maximum number of disjoint $A - B$ paths in G.

The minimum number of edges separating a from b in G is equal to the maximum number of edge disjoint $a - b$ paths in G.

A graph is k-edge connected if and only if it contains k edge disjoint paths between any two vertices.

Let G be a graph. Let $X \subseteq V(G)$. We say that X is linked in G if whenever we pick distinct vertices $s_1, s_2, \ldots, s_k, t_1, \ldots, t_k$ in X we can find disjoint paths $P_1, \ldots, P_k$ in G such that each P_i links s_i to t_i and has no inner vertex in X.
If $|V(G)| \geq 2k$, and every set of at most $2k$ vertices is linked in G, then G is k-linked.
Equivalently, a graph G is k linked if disjoint paths $P_1, \ldots, P_k$ (where P_i is from s_i to t_i) exist for every choice of exactly $2k$ vertices $s_1, s_2, \ldots, s_k, t_1, \ldots, t_k$