Graph Theory: Lecture No. 9

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$x - y$ **paths** P **and** Q **in** G **are internally disjoint if** $V(P) \cap V(Q) = \{x, y\}$**.**
The local connectivity $p(x, y)$ **between distinct vertices** x **and** y **is the maximum number of pairwise internally disjoint** $x - y$ **paths.**
$\kappa(G) = \min\{p(x, y) : x, y \in V(G), x \neq y\}$

Let x **and** y **be distinct non-adjacent vertices of** G**. An** xy**-vertex cut is a subset** S **of** $V - \{x, y\}$ **such that** x **and** y **belong to different components of** $G - S$**.**

Local edge connectivity between distinct vertices x and y is the maximum number of pairwise edge disjoint $x - y$ paths and is denoted by $p'(x, y)$.
A non-trivial graph G is k-edge connected if $p'(u, v) \geq k$ for any two distinct vertices u, v in G.
The edge connectivity $\kappa'(G)$ of a graph G is the maximum value of k for which G is k-edge connected.

$$\kappa \leq \kappa' \leq \delta$$

A maximal connected subgraph without a cut vertex is called a block.

The block graph of a connected graph is a tree.

If G is 3-connected and $|G| > 4$ then G has an edge e such that G/e is again 3-connected.

A graph G is 3-connected if and only if there exists a sequence $G_0, G_1, \ldots, G_n$ of graph such that the following properties hold.
(1) $G_0 = K_4$ and $G_n = G$.
(2) G_{i+1} has an edge (x, y) with $d(x), d(y) \geq 3$ and $G_i = G_{i+1}/(x, y)$, for every $i < n$.