# Graph Theory: Lecture No. 13

L. Sunil Chandran

Computer Science and Automation,
Indian Institute of Science, Bangalore
Email: sunil@csa.iisc.ernet.in

**A vertex coloring of a graph $G = (V, E)$ is a map $c : V \to S$ such that $c(v) \neq c(w)$ whenever $v$ and $w$ are adjacent. Let elements of the set are called the available colors. The smallest integer $k$ such that $G$ has a $k$-coloring i.e. a vertex coloring $c : V \to \{1, \ldots, k\}$ is called the chromatic number $\chi(G)$ of the graph $G$.**

**Every graph with $m$ edges satisfies**

$$\chi(G) \leq \frac{1}{2} + \sqrt{2m + \frac{1}{4}}$$

**The least number $k$ such that $G$ has a vertex enumeration in which each vertex is preceded by fewer than $k$ of its neighbors is called the coloring number $col(G)$ of $G$.**

$$\chi(G) \leq col(G) = \max\{\delta(H) | H \subseteq G\} + 1$$

**Let $G$ be a connected graph. If $G$ is neither complete nor an odd cycle, then:**

$$\chi(G) \leq \Delta(G)$$