Graph Theory: Lecture No. 14

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(Brooks 1941) Let G be a connected graph. If G is neither complete nor and odd cycle, then $\chi(G) \leq \Delta(G)$.

For every integer k, there exists a graph G with girth $g(G) > k$ and chromatic number $\chi(G) > k$.

The class of k-constructible graphs:

1. **K_k is k-constructible.**
2. **If G is k-constructible, and $x, y \in V(G)$ are non-adjacent, then also $(G + xy)/xy$ is k-constructible.**
3. **If G_1, G_2 are k-constructible and there are vertices x, y_1, y_2 such that $G_1 \cap G_2 = \{x\}$ and $xy_1 \in E(G_1)$ and $xy_2 \in E(G_2)$ then also $(G_1 \cup G_2) - xy_1 - xy_2 + y_1y_2$ is k-constructible.**

Let G be a graph and $k \in N$. Then $\chi(G) \geq k$ if and only if G has a k-constructible subgraph.