Graph Theory: Lecture No. 15

L. Sunil Chandran

Computer Science and Automation,
Indian Institute of Science, Bangalore
Email: sunil@csa.iisc.ernet.in

The class of k-constructible graphs:

1. **K_k is k-constructible.**
2. **If G is k-constructible, and $x, y \in V(G)$ are non-adjacent, then also $(G + xy)/xy$ is k-constructible.**
3. **If G_1, G_2 are k-constructible and there are vertices x, y_1, y_2 such that $G_1 \cap G_2 = \{x\}$ and $xy_1 \in E(G_1)$ and $xy_2 \in E(G_2)$ then also $(G_1 \cup G_2) - xy_1 - xy_2 + y_1y_2$ is k-constructible.**

Let G be a graph and $k \in N$. Then $\chi(G) \geq k$ if and only if G has a k-constructible subgraph.

Given a positive integer k, we can construct triangle-free graphs with $\chi(G) \geq k$.

A proper edge coloring of G is a coloring of edges of G such that no two adjacent edges get the same color.
The minimum number of colors required for properly edge coloring a graph G is known as its chromatic index $\chi'(G)$.

Every bipartite graph G satisfies $\chi'(G) = \Delta(G)$.