Graph Theory: Lecture No. 18

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Kuratowsky's Theorem: The following assertions are equivalent for graphs G:

(1) G is planar

(2) G contains neither K_5 nor $K_{3,3}$ as a topological minor.

(3) G contains neither $K_{3,3}$ or K_5 as a minor

Let $G = (V, E)$ and S_v, $v \in V$ a family of sets. We call a vertex coloring $c(v) \in S_v$ for all $v \in V$ a colouring from the lists S_v. The graph G is called k-list colourable or k-choosable if for every family $(S_v), v \in V$ with $|S_v| = k$ for all $v \in V$, there is a (proper) vertex coloring of G. The least integer k for which G is k-choosable is called the choice number of G (or the list chromatic number) of G.

$$ch(G) \geq \chi(G)$$

Every planar graph is 5-choosable.

Suppose that every inner face of G is bounded by a triangle and its outer face by a cycle $C = \{v_1, v_2, \ldots, v_k, v_1\}$. Suppose further that v_1 has already been coloured with the colour 1, and v_2 has been coloured with 2. Suppose finally that with every other vertex of C a list of at least 3 colours is associated and with every vertex of $G - C$ a list of at least 5 colours. Then the colouring of v_1 and v_2 can be extended to a colouring of G from the given lists.