Graph Theory: Lecture No. 19

L. Sunil Chandran

Computer Science and Automation,
Indian Institute of Science, Bangalore
Email: sunil@csa.iisc.ernet.in

The least integer k **such that** G **has an edge coloring from any family of lists of size** k **is the list chromatic index** $ch'(G)$ **of** G**. That is** $ch'(G) = ch(L(G))$ **where** $L(G)$ **is the line graph of** G**. Clearly** $ch'(G) \geq \chi'(G)$

The List Coloring Conjecture: Every graph G satisfies $ch'(G) = \chi'(G)$.

Let D be a directed graph. An independent set $U \subseteq V(D)$, such that for every vertex $v \in D - U$, there is an edge in D directed from v to a vertex in U, is called a kernel of D.

Let H be a graph and $(S_v)_{v \in V(H)}$ be a family of lists. If H has an orientation D with $d^+(v) < |S_v|$ for every vertex v and such that every induced subgraph of D has a kernel, then H can be colored from the list S_v.

Let a family $(\leq_v)_{v \in V}$ of linear orderings $\leq_v$ on $E(v)$ a set of preferences for G. Then call a matching M in G stable if for every edge $e \in E - M$, there exists an edge $f \in M$ such that e and f have a common vertex v with $e \leq_v f$.

For every set of preferences, G has a stable matching.

Every bipartite graph G satisfies,
$ch'(G) = \chi'(G)$.

A matching M in G is better than a matching $M' \neq M$ if M makes the vertices in B happier than M' does, i.e. if every vertex b in an edge $f' \in M'$ is incident also with some $f \in M$ such that $f' \leq_b f$.

Given a matching M, call a vertex $a \in A$ acceptable to $b \in B$ if $e = ab \in E - M$ and any edge $f \in M$ at b satisfies $f \leq_b e$.

$a \in A$ **is happy with** M **if** a **is unmatched or its matching edge** $f \in M$ **satisfies** $f >_a e$ **for all edges** $e = ab$ **such that** a **is acceptable to** b**.**