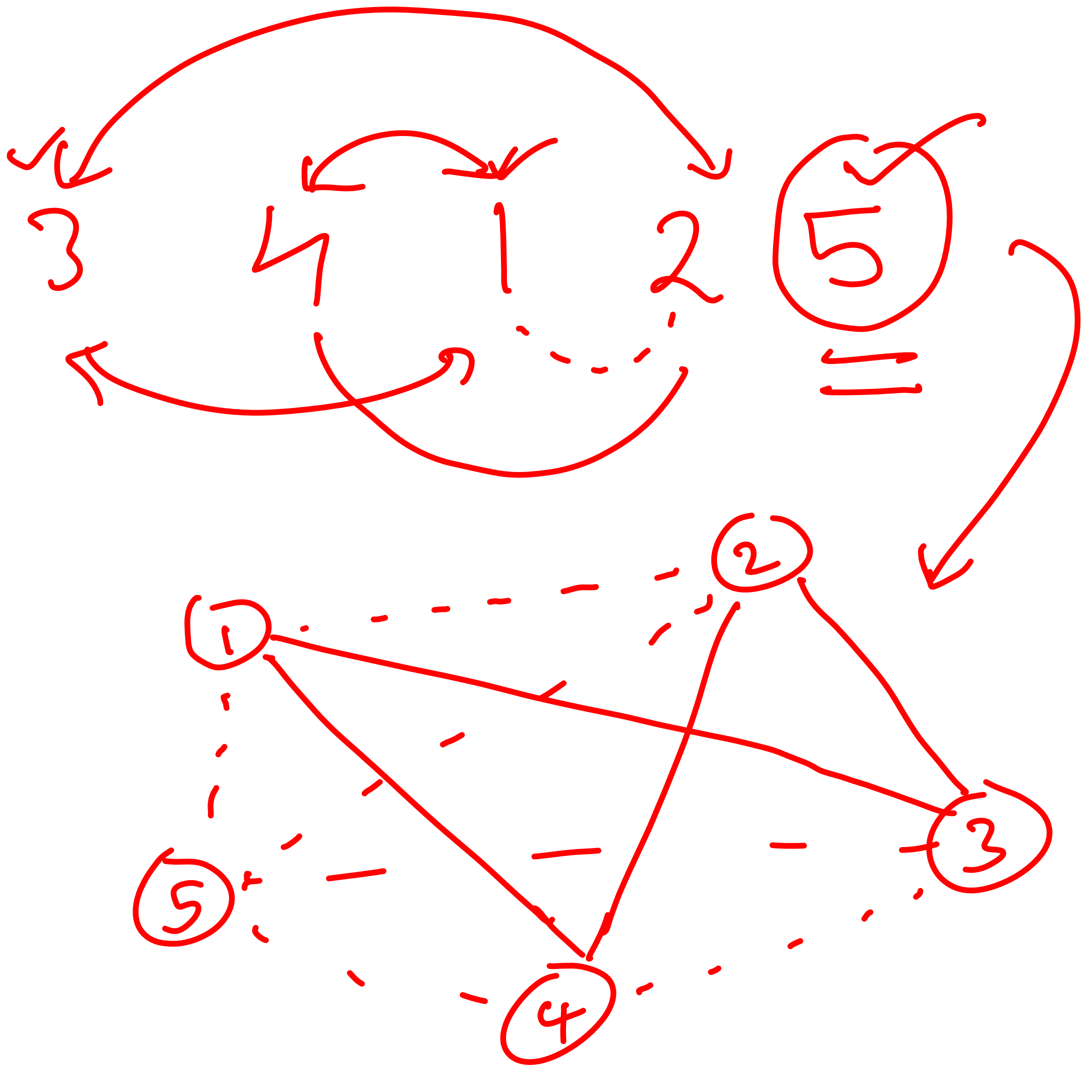
Graph Theory-

Lecture No: 27

Permutation graphs

1	2	3	4	5	6
1	3	2	5	6	4
3	4	5	1	2	6



$i \quad j$

$i < j$

$1 < 3$

1 2 3 4 5

02

1

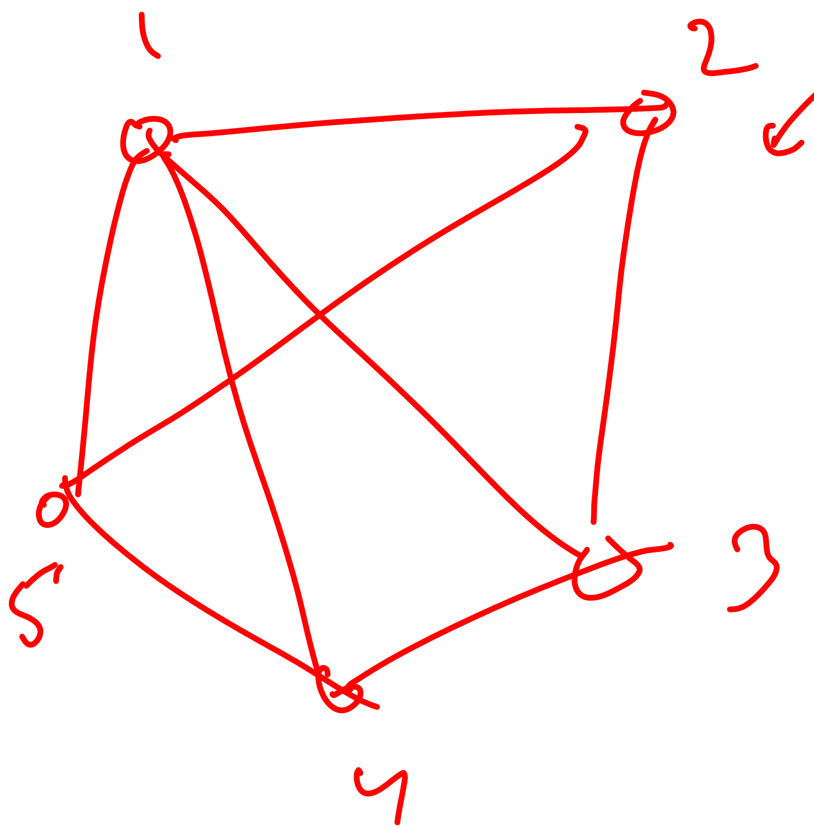
0

03

0

5 04

5 4 1 2 3



i j

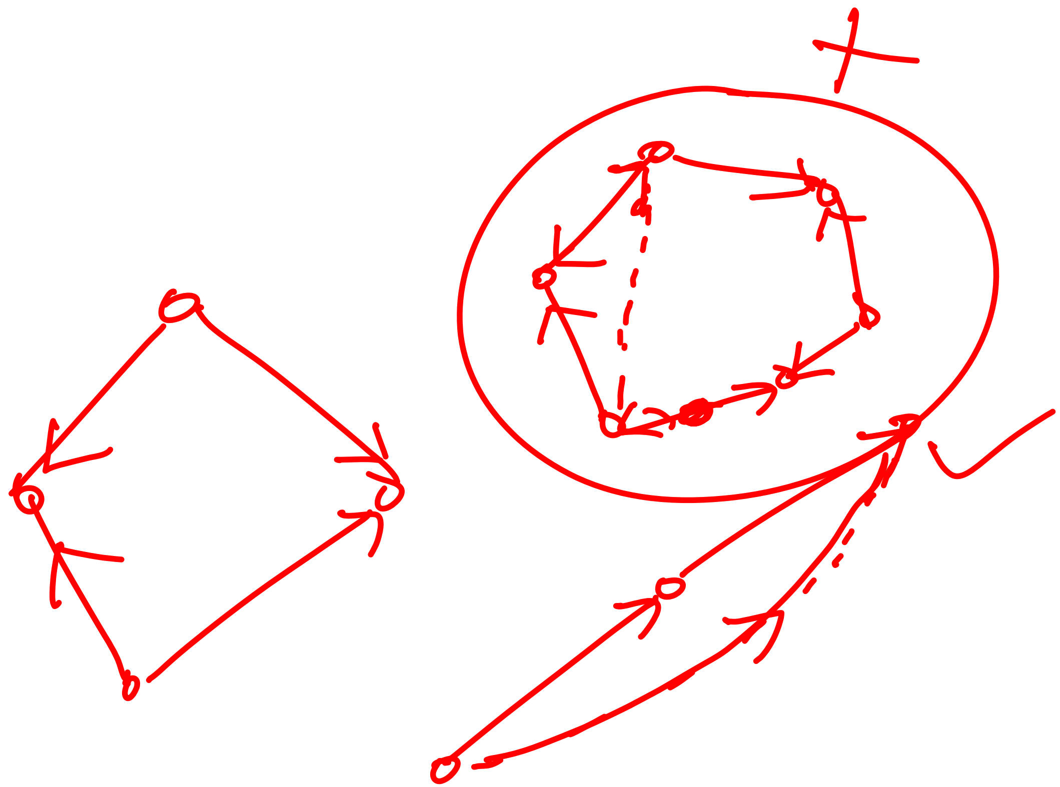
0 i

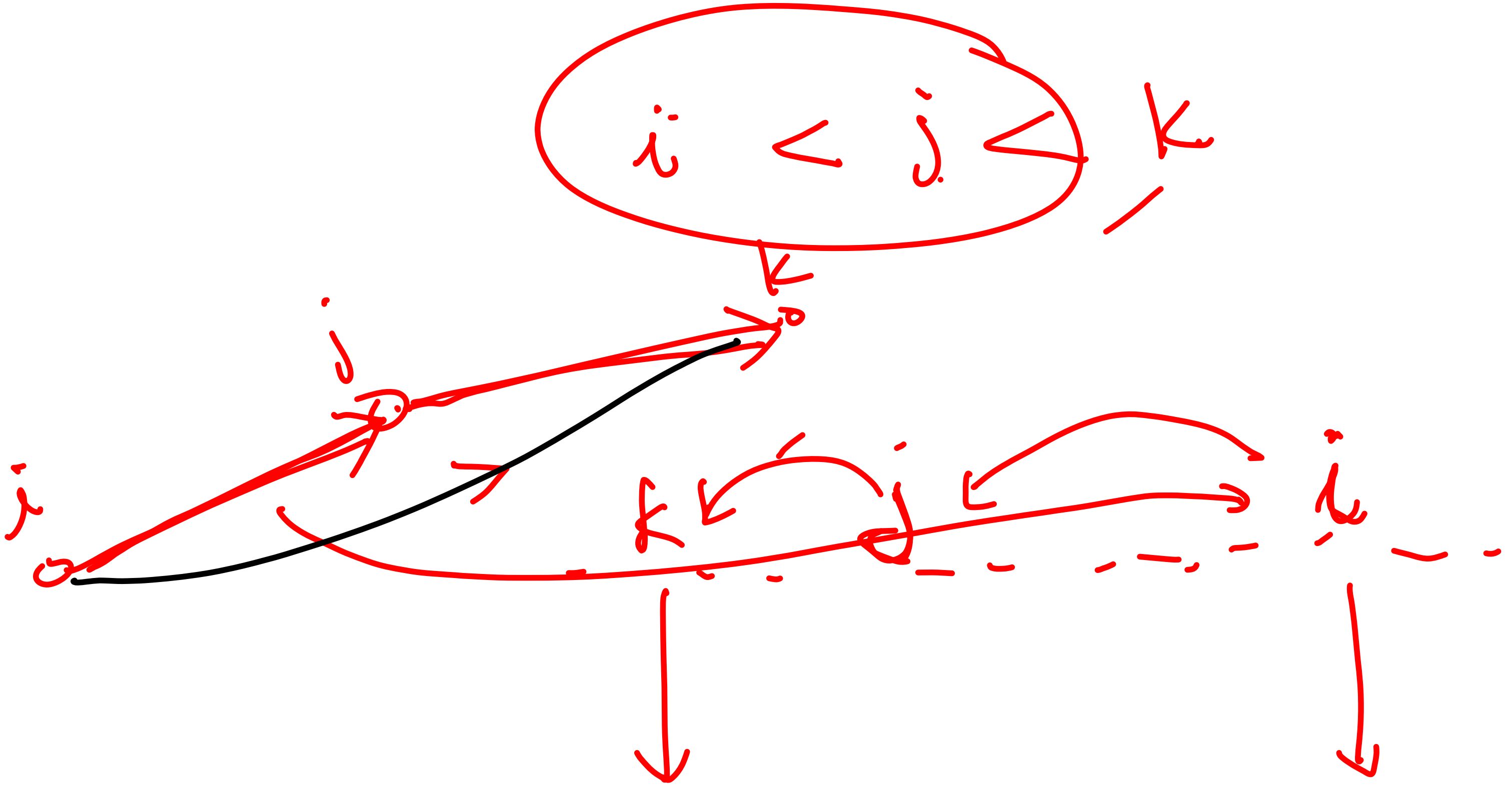
i . . . j ✓

i . . . j

A permutation graph is a graph which is isomorphic to ~~some~~ $G(\pi)$ for some permutation π

Why is a permutation graph a comparability graph?





i < j < k
k
j
i
k
j
i

A graph G is a permutation graph if and only if G and $\overline{G}$ are comparability graph.

$$P = \mathrm{Comp.} \cap \cancel{\quad} \text{co-Comp}$$

$G(\pi) \longrightarrow$

$G(\pi_R) \longrightarrow$

G ✓ is a permutation graph $\longrightarrow$

$\overline{G}$ ✓ " $\longrightarrow$

$G \longrightarrow$ comparability graph

$\overline{G} \longrightarrow$ "

G is a permutation graph?

$$G' \longrightarrow \vec{F}$$ ✓

$$\bar{G} \longrightarrow \vec{F}'$$

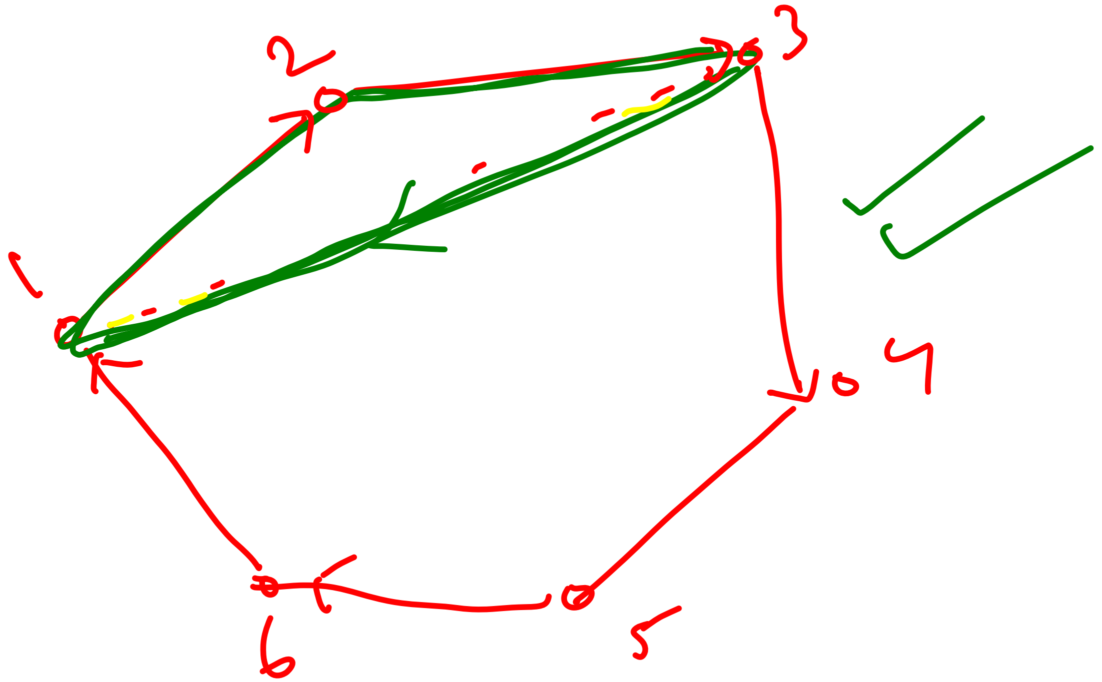
$$G \cup \bar{G}$$

$$\downarrow$$

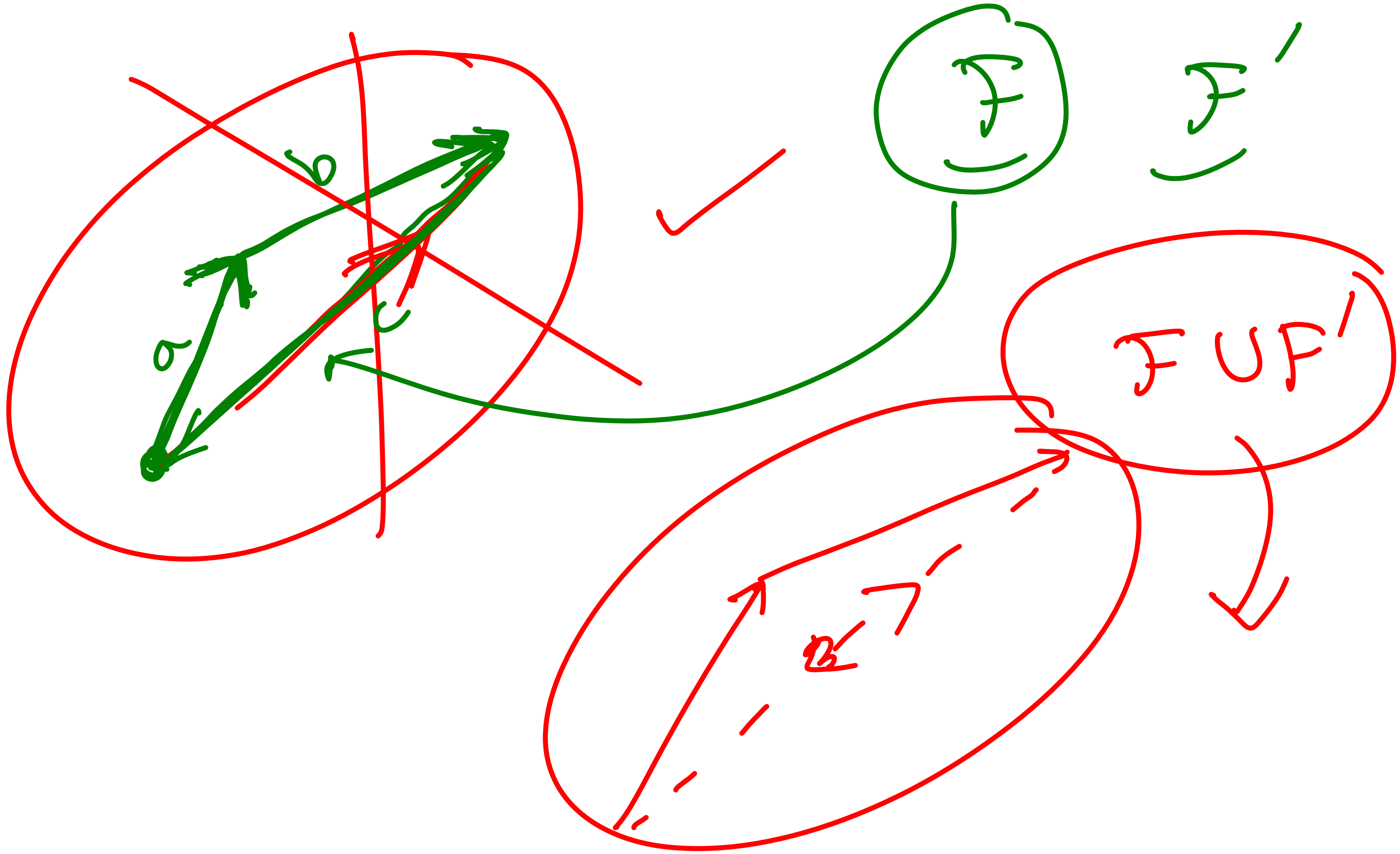
$$K_n$$

$$\vec{F} \cup \vec{F}'$$ ✓

$\vec{F} \cup \vec{F}'$ ✓ for the edge of K_n ?



$k > 3$


a
b
c
F
F'
F ∪ F'
B

$G \longrightarrow \mathcal{F} \quad \bar{\mathcal{F}}$

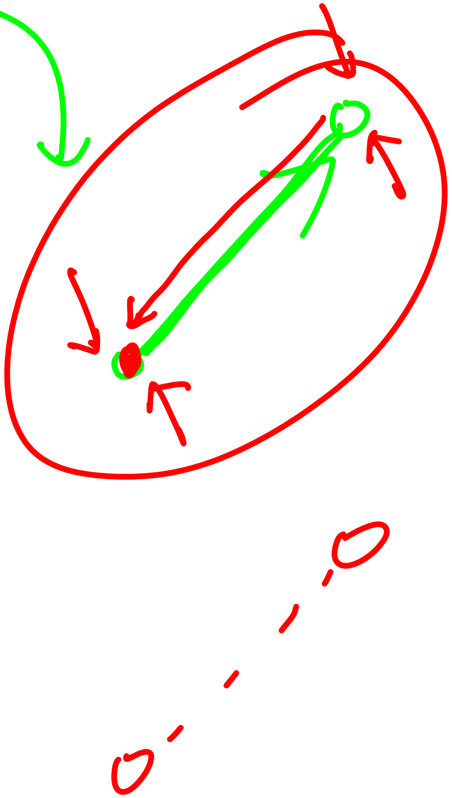
$\bar{G} \longrightarrow \mathcal{F}'$

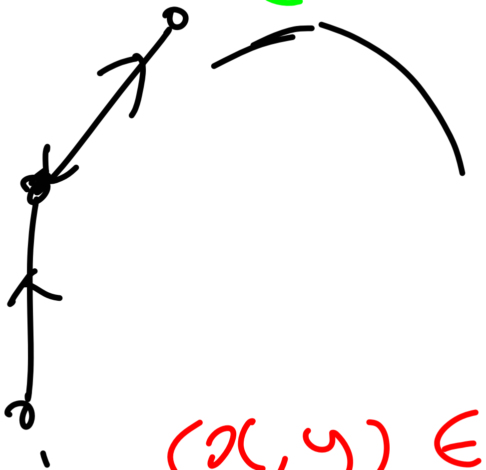
$\mathcal{F} \cup \mathcal{F}'$ ✓

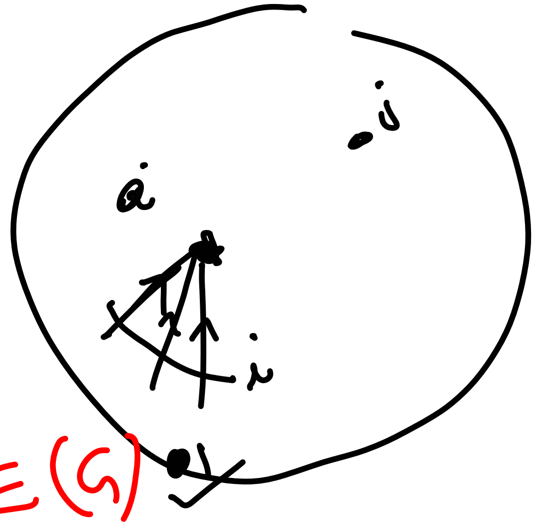
$\bar{\mathcal{F}} \cup \mathcal{F}' \longrightarrow K_n$

$F \cup F' \longrightarrow h(x) = i+1$

$\bar{F} \cup F' \longrightarrow h'(x)$

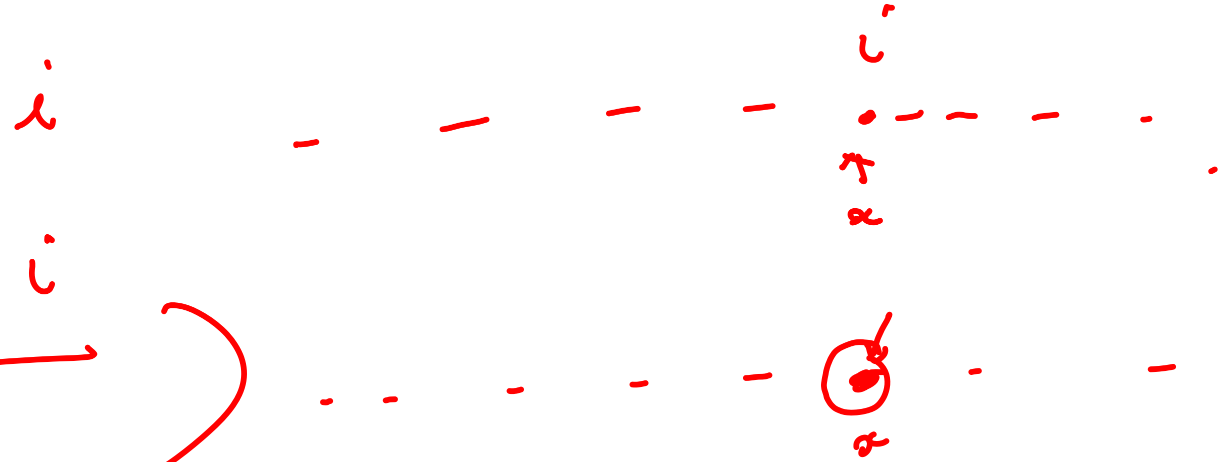






$(x,y) \in E(G)$

$$\left[h(x) - h(y)\right]\left[h'(x) - h'(y)\right] \le r$$



if $h(x) = i$

$$\pi^{-1}_i = h'(x) \qquad (i, j) \in E$$

$$[h(x) - h(y)]\,[h'(x) - h'(y)] < 0$$

$$(i - j)\,(\pi^{-1}_i - \pi^{-1}_j) < 0$$

$$F \cup F' \longrightarrow \quad x \to y, \quad h(x) < h(y)$$

$$\bar{F} \cup P' \longrightarrow \quad x \to y, \quad h'(x) < h'(y)$$

$$\boxed{[h(x) - h(y)]\,(h'(x) - h'(y)) > 0}$$

$$\underbrace{\acute{F}\,\mathrm{UP}'}\longrightarrow \underbrace{h(x)}$$

$$\acute{F}\,\mathrm{UP}' \longrightarrow h'(x)$$

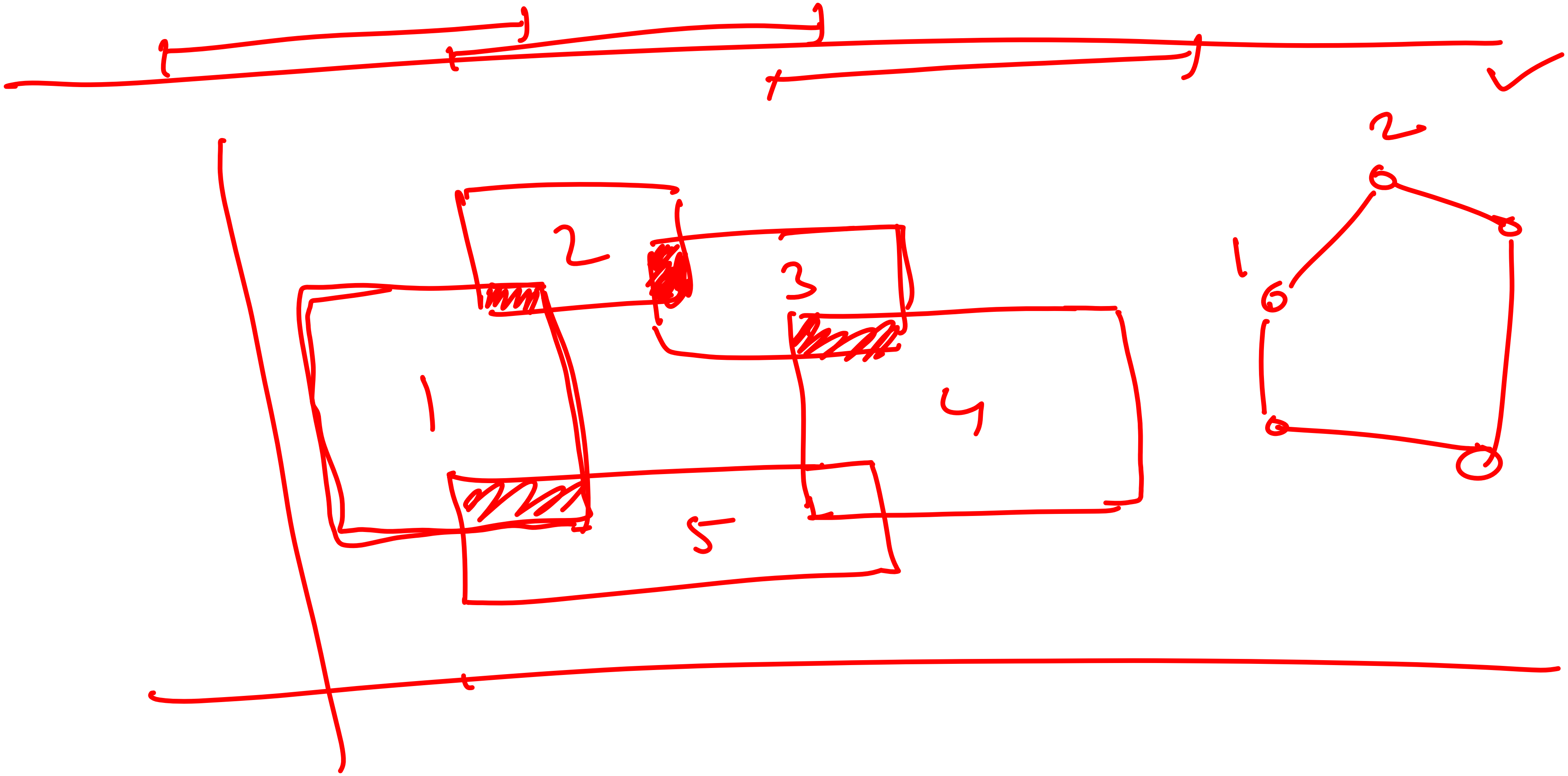
$$\underline{[h(x)-h(y)][h'(x)-h'(y)]<0}$$

i

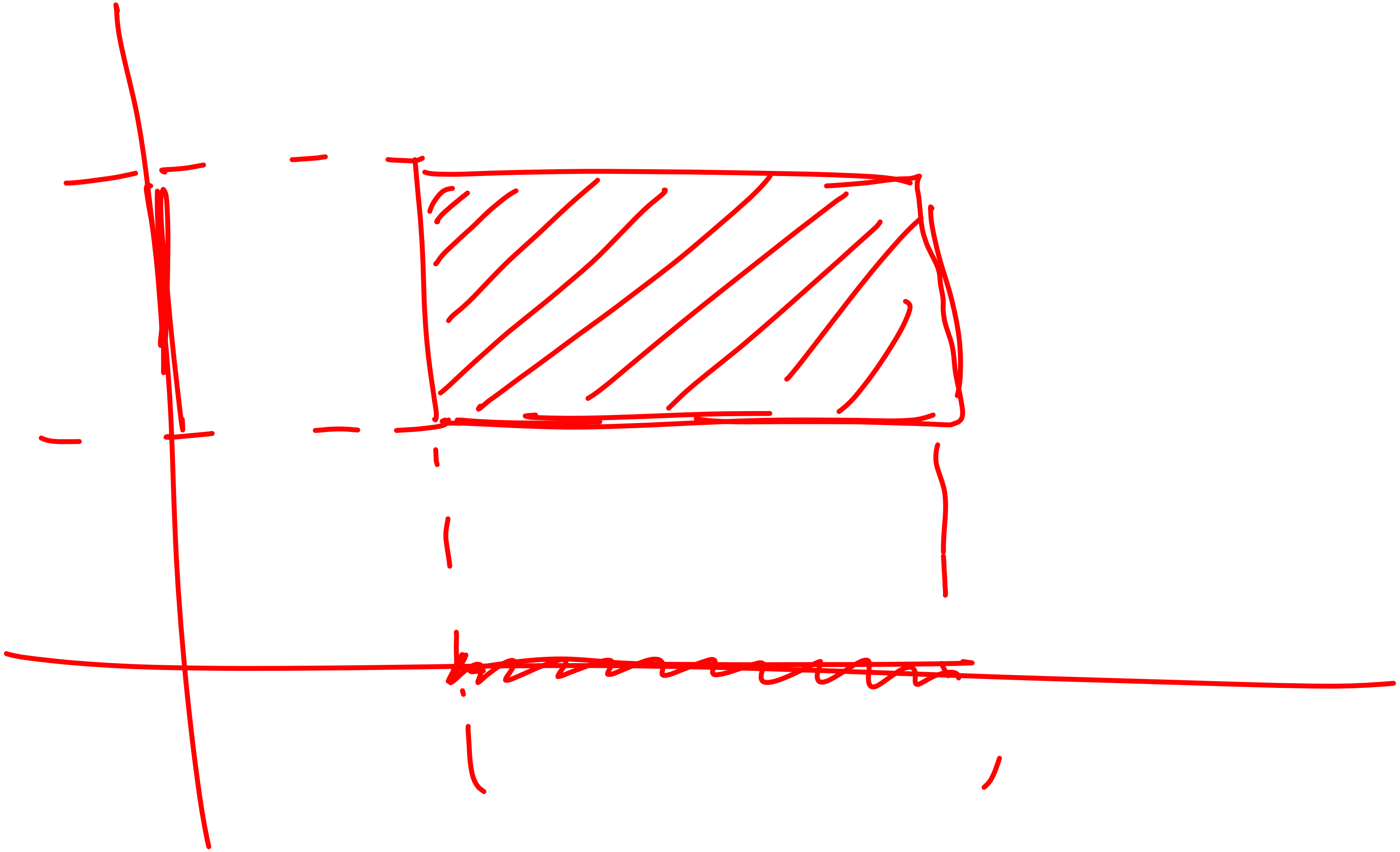
$$h(x) = i$$

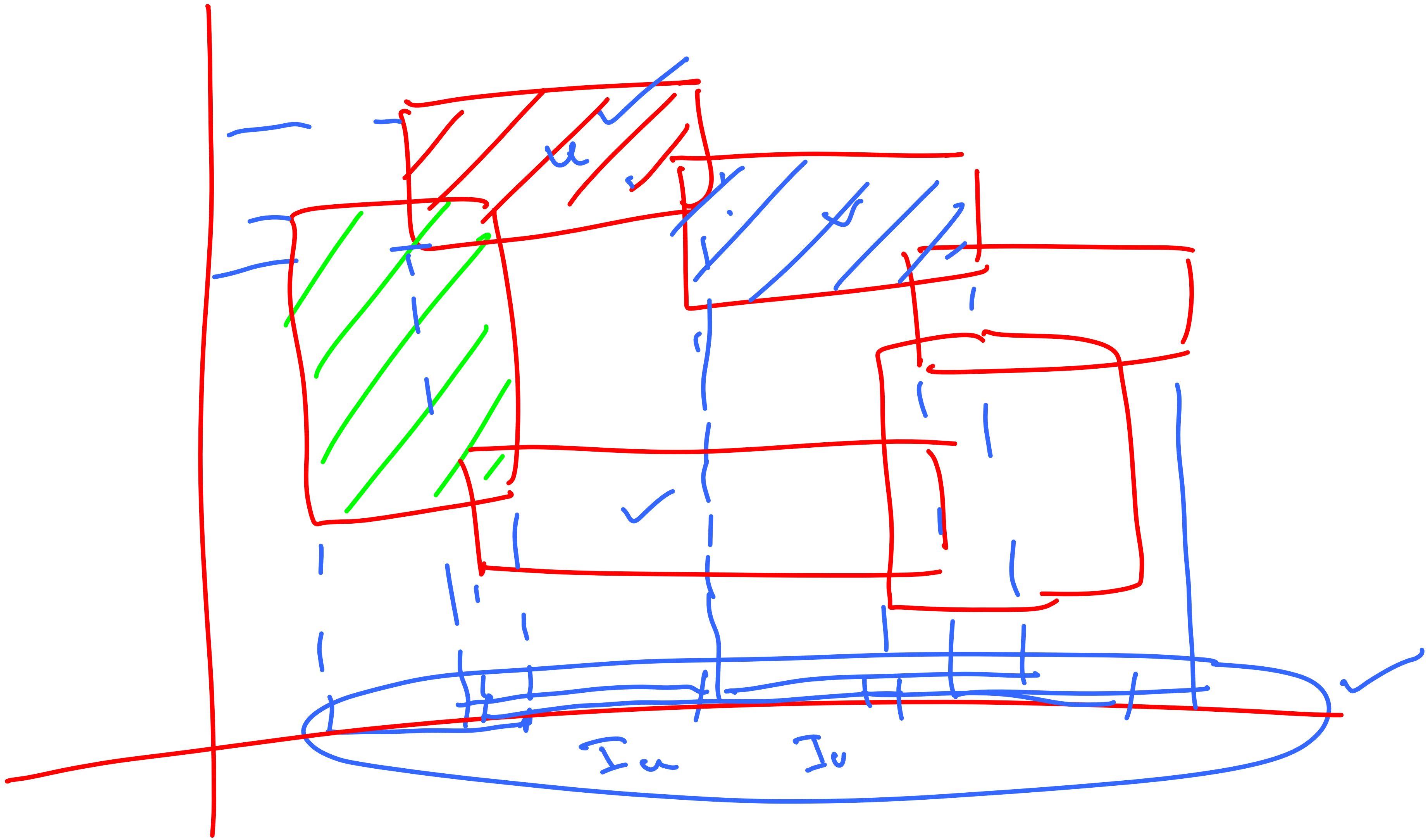
$$\pi_i^{-1} = h'(x)$$

$$\boxed{(\pi_i^{-1} - \pi_j^{-1})(i-j) < 0} \checkmark$$



2
3
1
4
5
2
1





u
v
I_u
I_v

G_1 is the interval super graph obtained by taking projection to x-axis

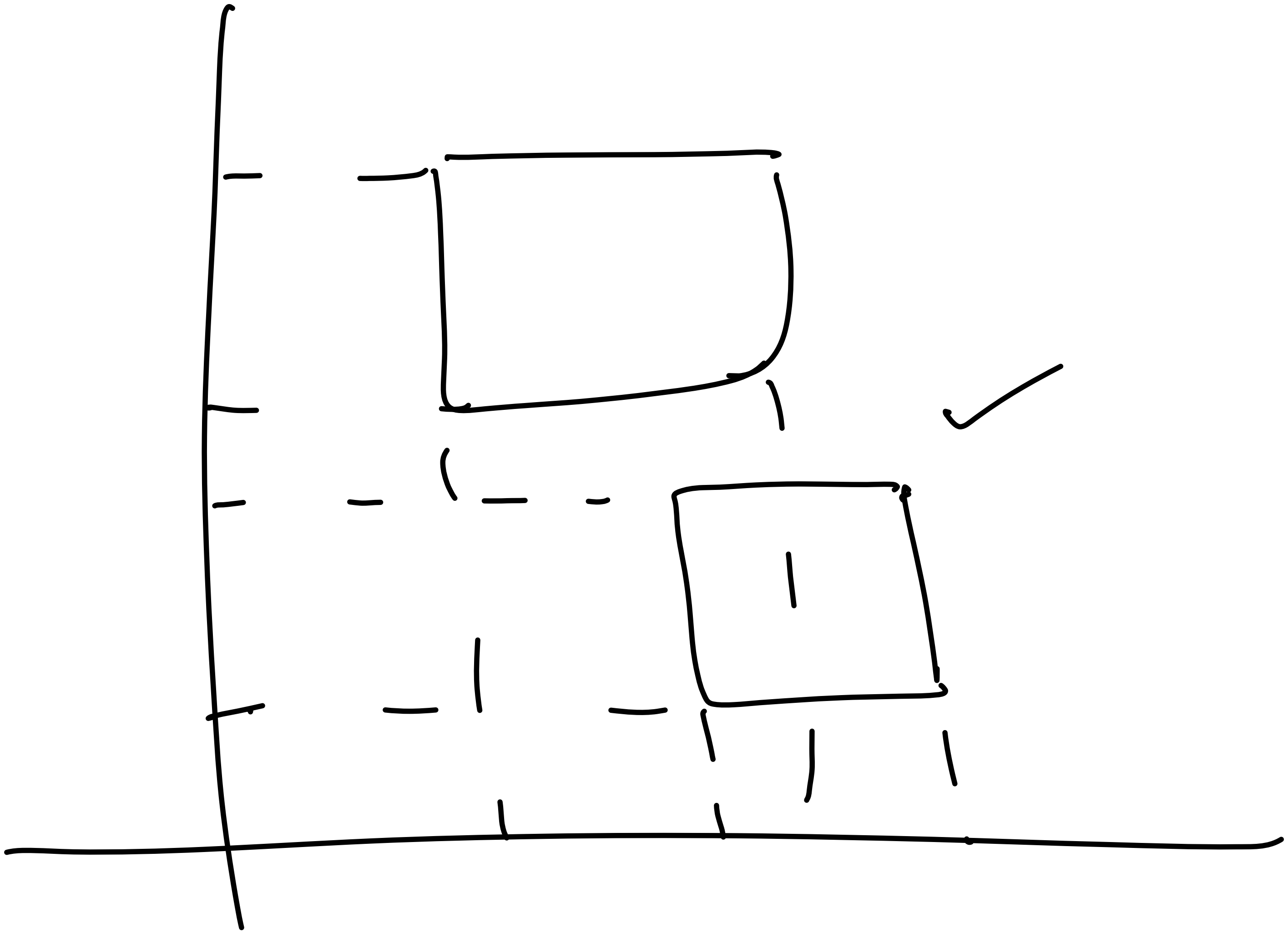
G_2 - - - to y-axis.

$$V(G_1) = V(G_2) = V(G)$$

$$\boxed{E(G) = \underbrace{E(G_1)}\cap\underbrace{E(G_n)}}$$

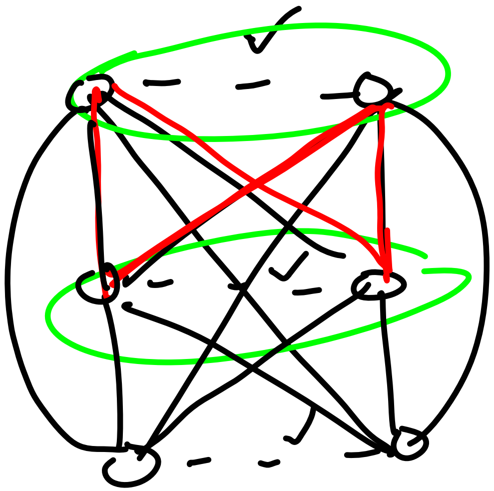
$$E(G_1) \supseteq E(G)$$

$$E(G_2) \supseteq E(G)$$



I_1 and I_2

$G = I_1 \cap I_2$



I_1 or I_2