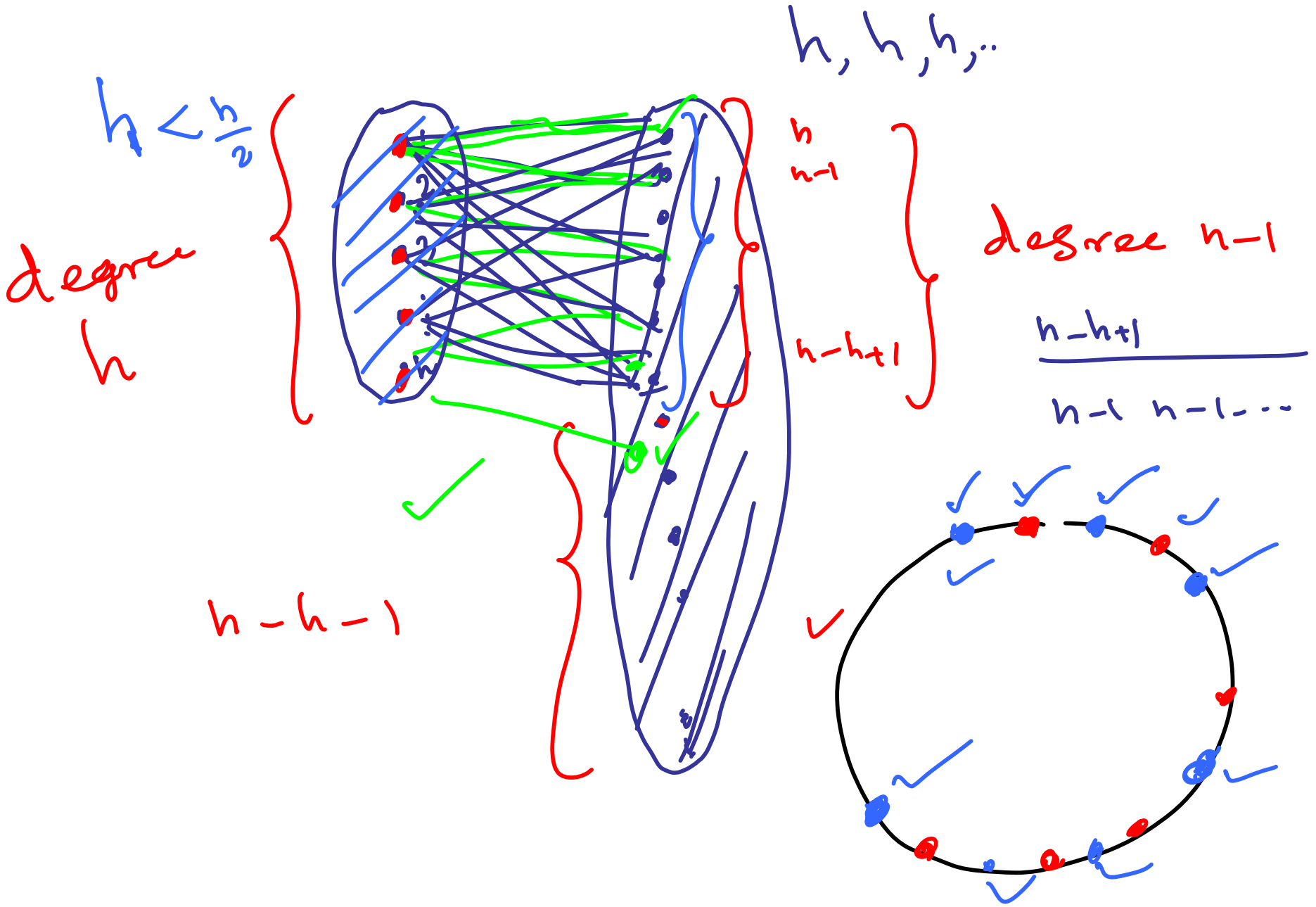
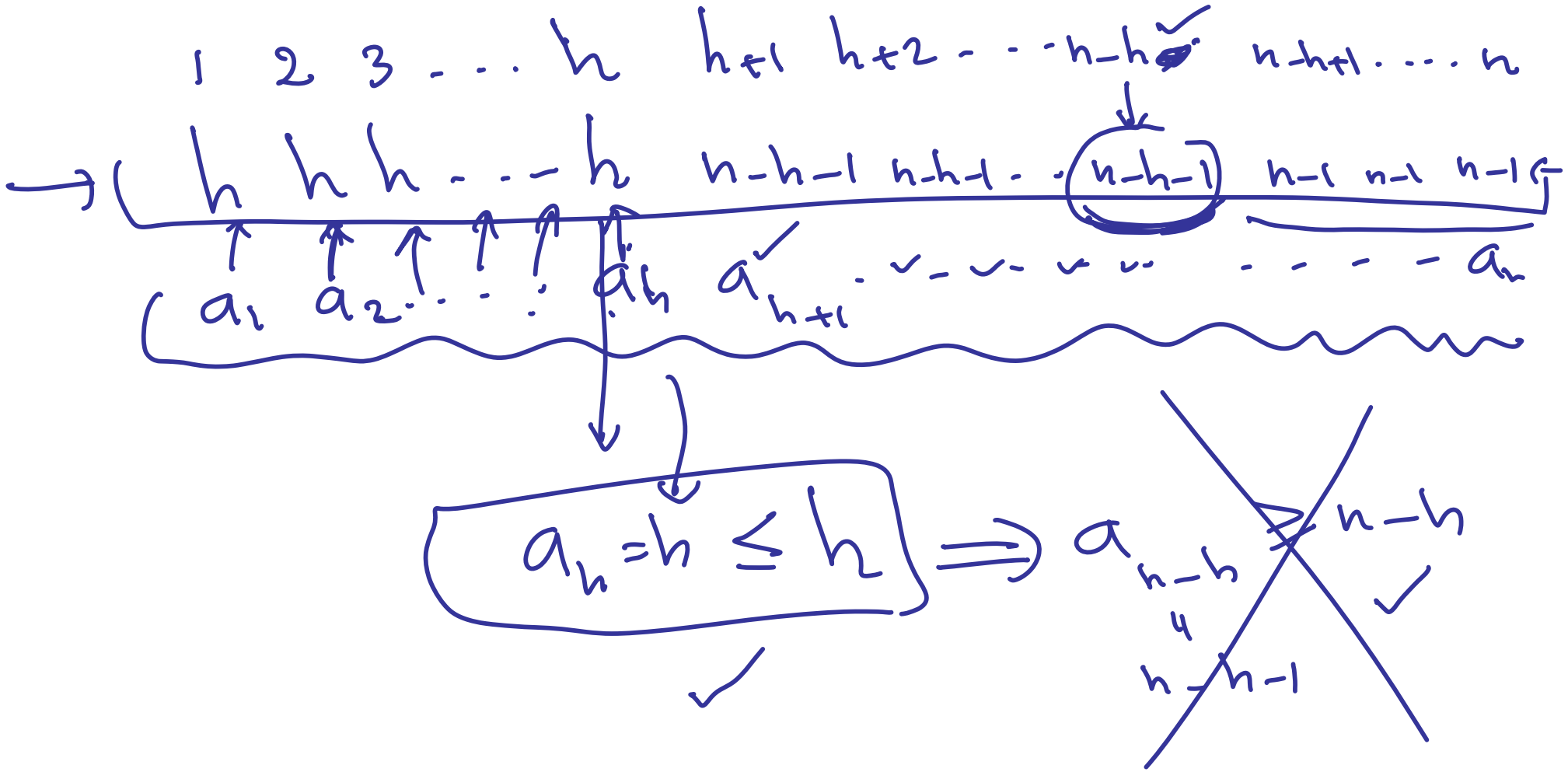
$k < \frac{n}{2}$

$a_k \leq k$, but

$a_{n-k} \leq n-k$.


h, h, h, ..
h < h/2
degree h
h
h-1
h-h+1
degree h-1
h-h+1
h-h-1
h-1 h-1 ...


1 2 3 ... h h+1 h+2 ... n-h n-h+1 .... n
h h h ... h n-h-1 n-h-1 ... n-h-1 n-1 n-1 n-1
a1 a2 ... ah ah+1 ... an
ah = h ≤ h
an-h ≥ n-h
4
n-h-1

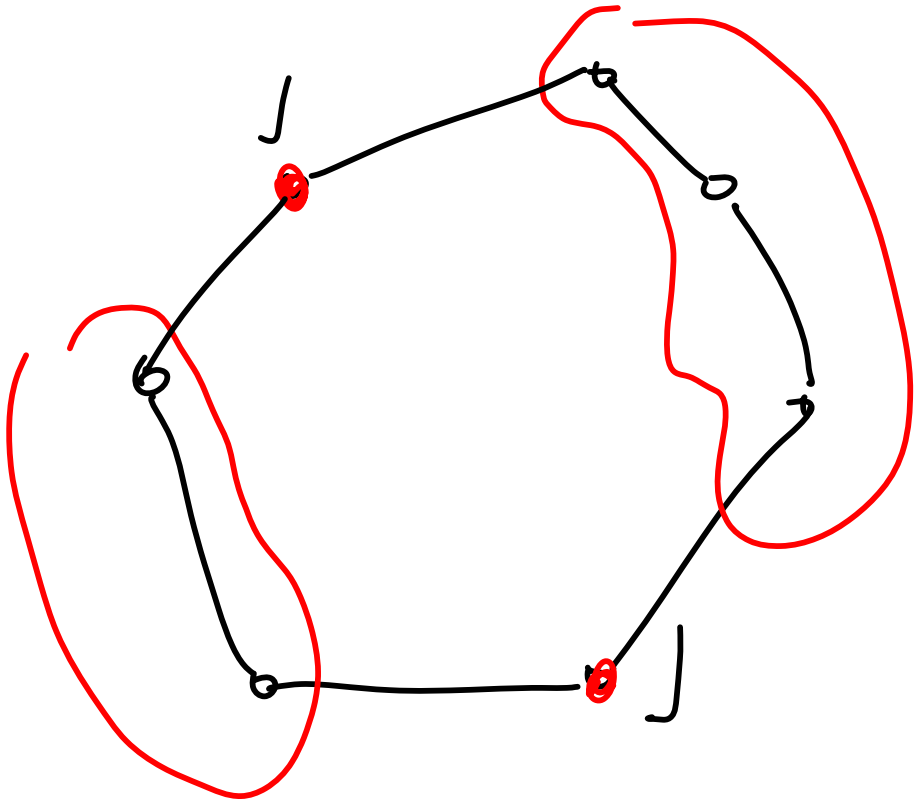
toughness of a graph

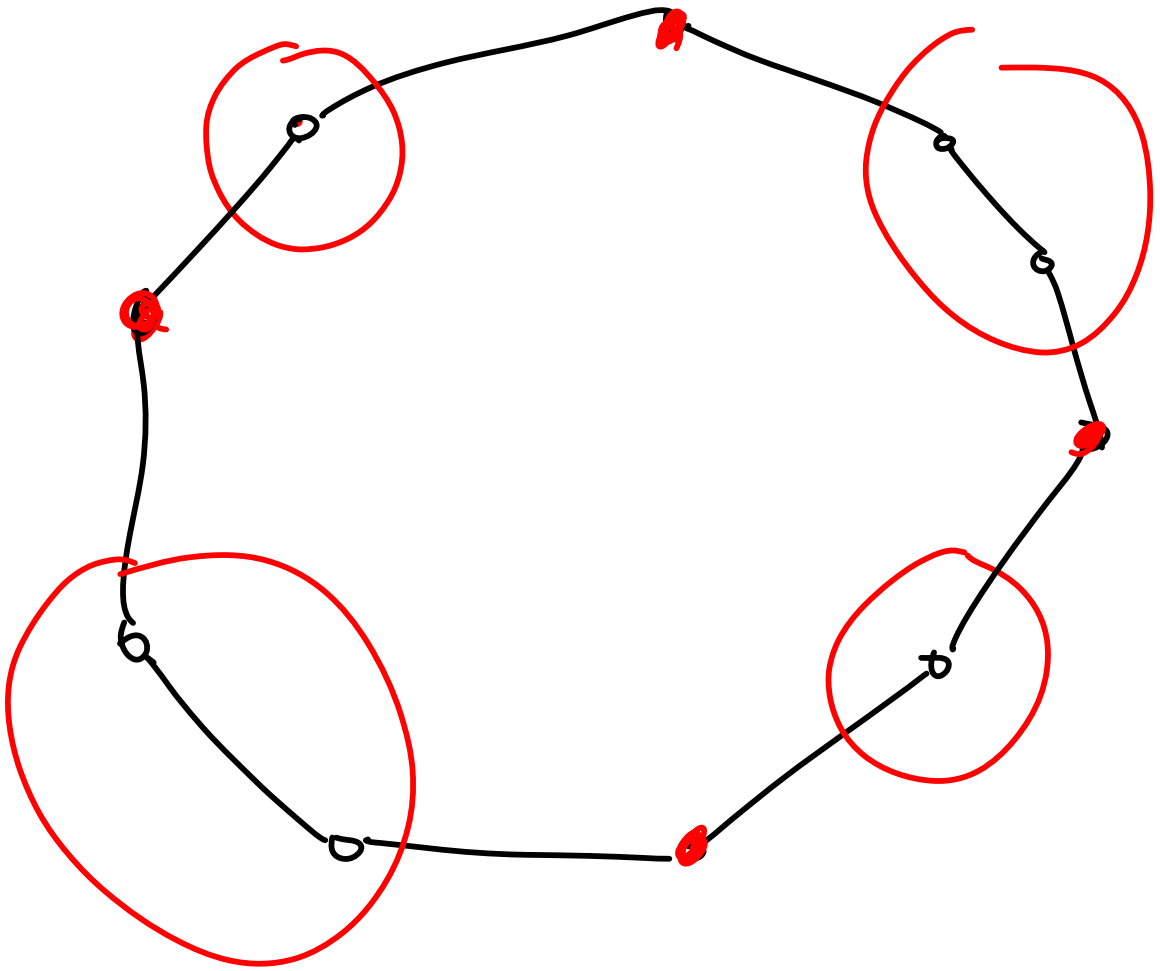
remove "S" vertices from G

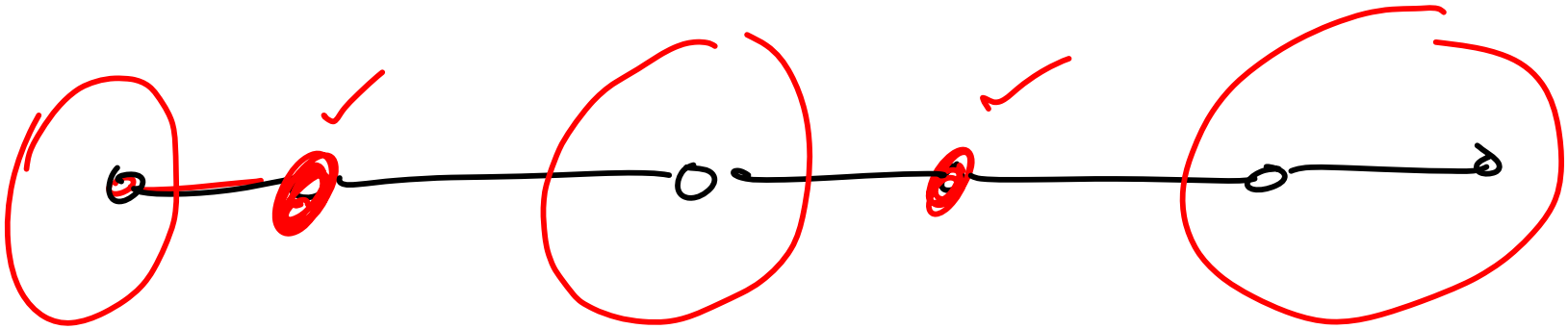
$$\#\text{components in } G - S \leq |S|$$

then G is tough

1-tough







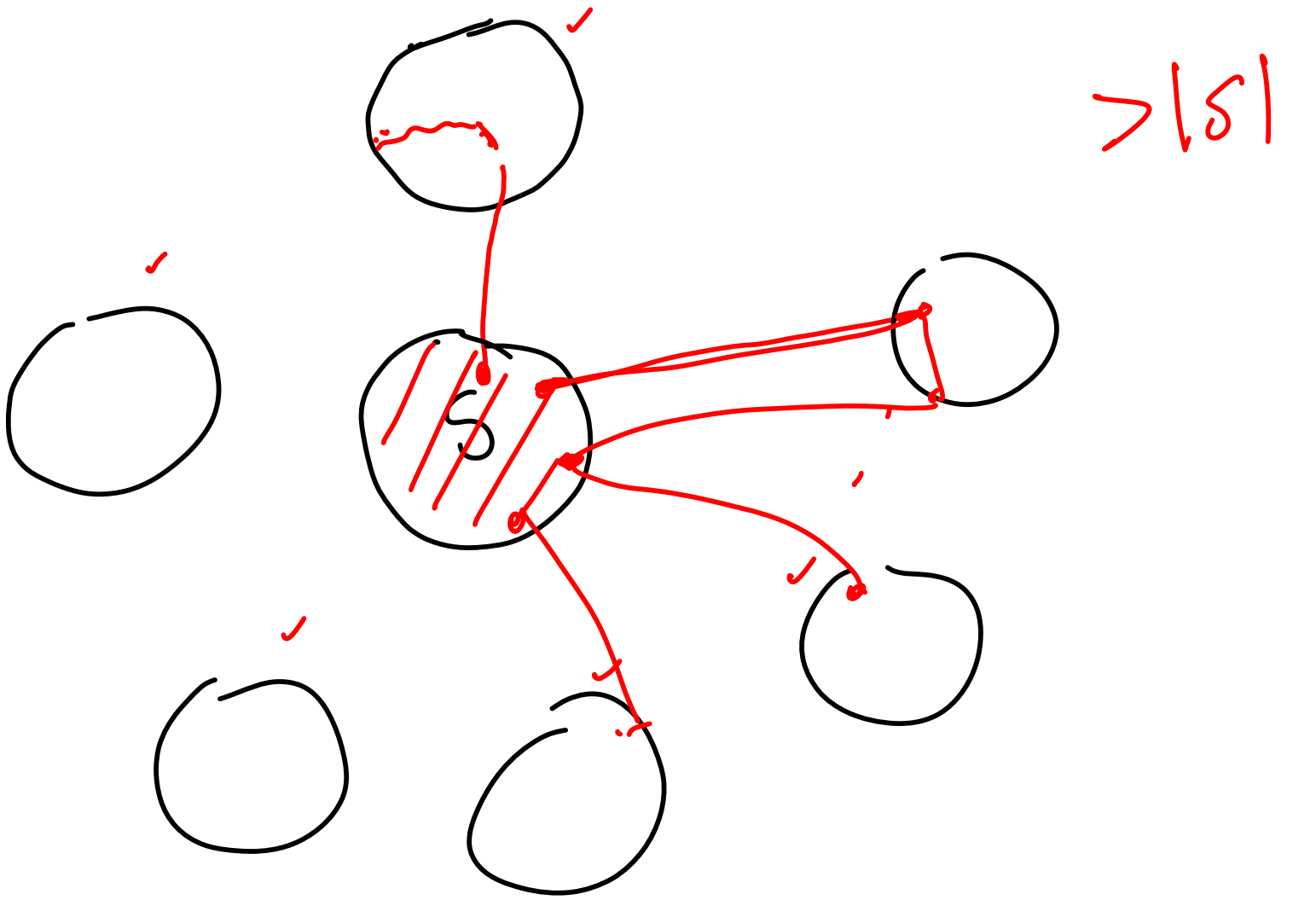
$t > 0$ be a real number

then

$$S$$

$$\#\text{components in } G - S \leq \frac{|S|}{t}$$

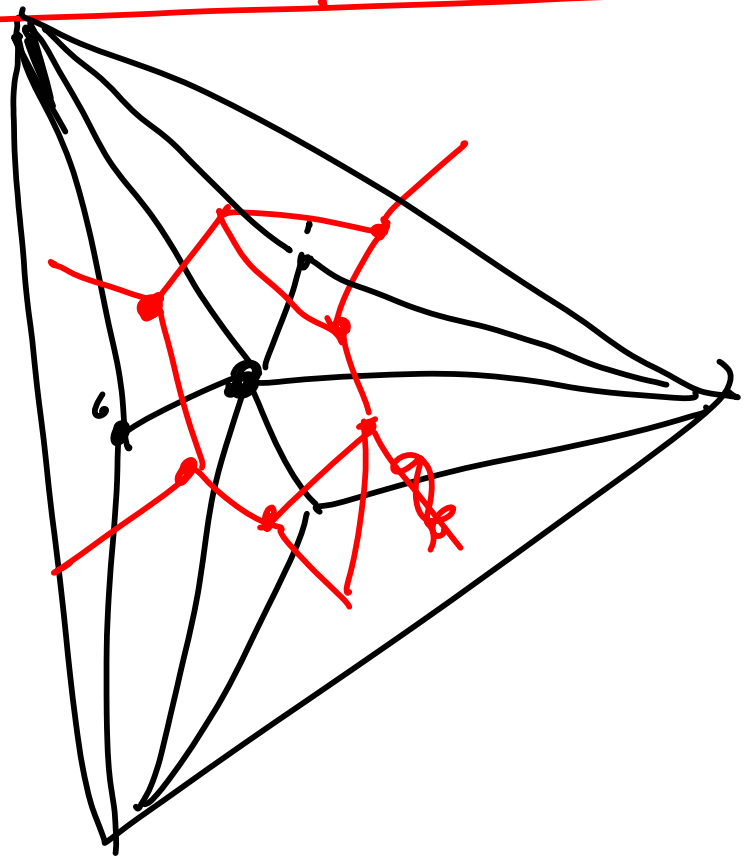
G is t-tough

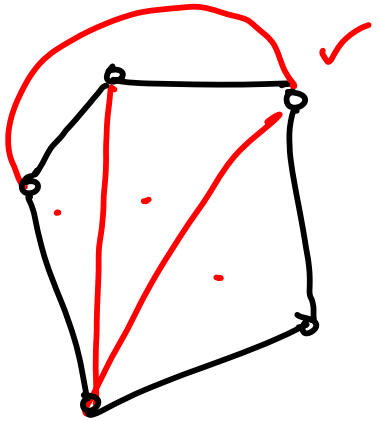

S
>|S|

3 - connected maximal planar graphs

*triangulation

"3"

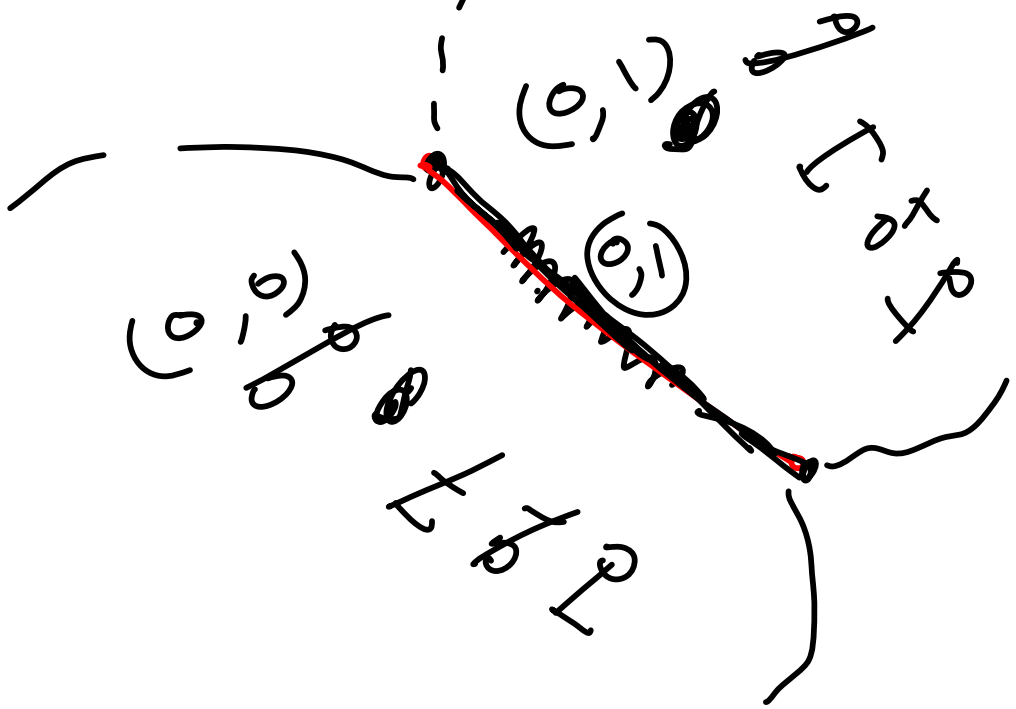




0, 1, 2, 3 ✓

(0,0) (0,1) (1,0) (1,1)

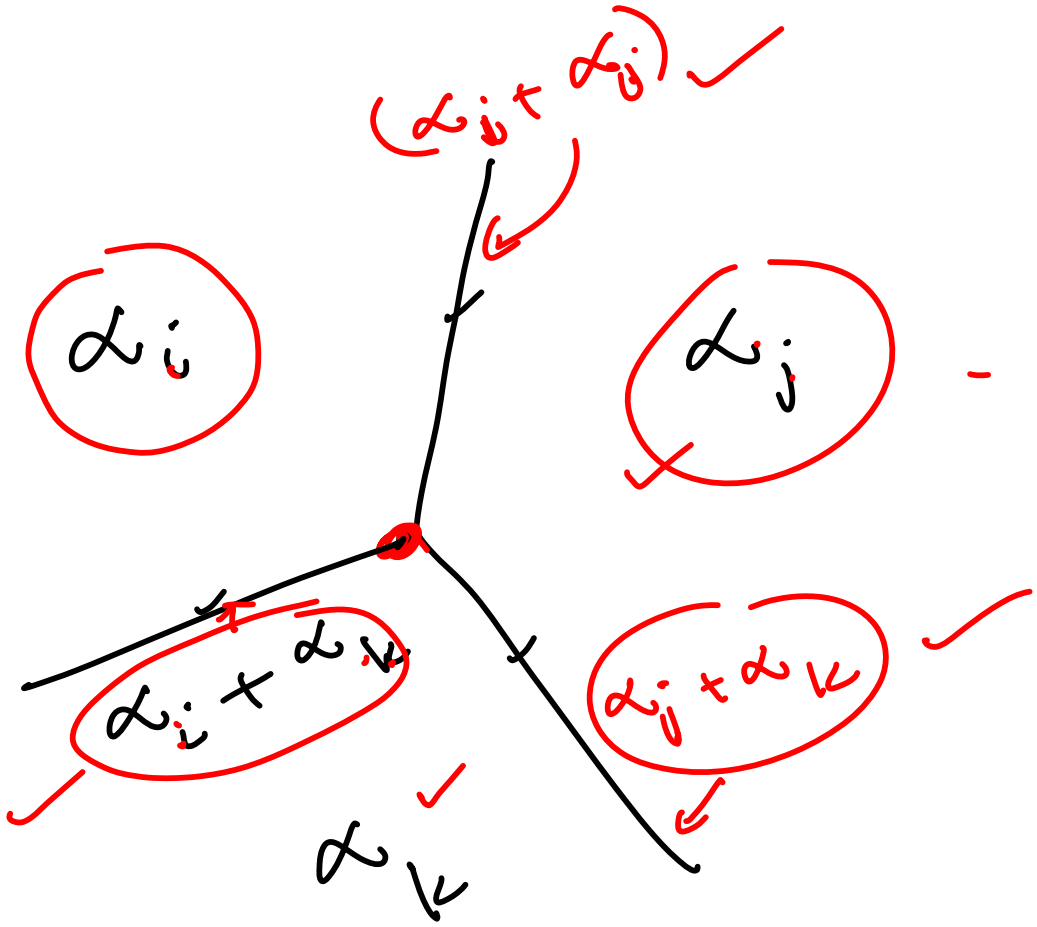
α_0 α_1 α_2 α_3

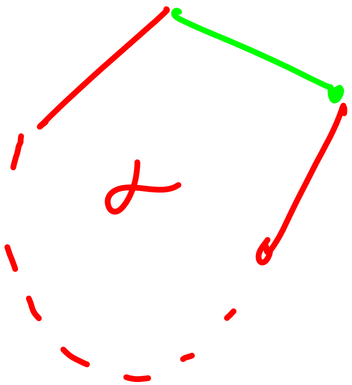


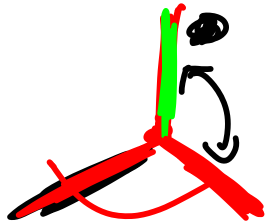
(0,0) +
(0,1)

(0,1

(0,0)

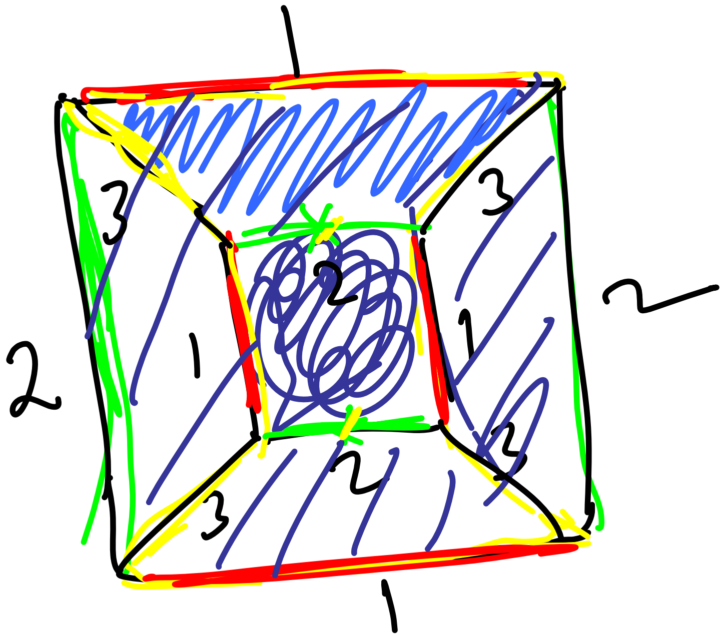

$(\alpha_i + \alpha_j)$
α_i
α_j
$\alpha_i + \alpha_k$
$\alpha_j + \alpha_k$
α_k





(0, 1) 0 or

(0, 1) 1



0

1