Graph Theory: Lecture No. 30

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If G is a graph with n vertices and degrees $d_1 \leq d_2 \leq \ldots \leq d_n$ then the n-tuple $(d_1, \ldots, d_n)$ is called the degree sequence of G.
An arbitrary integer sequence $(a_1, a_2, \ldots, a_n)$ is called Hamiltonian, if every graph with n vertices and a degree sequence pointwise greater than $(a_1, a_2, \ldots, a_n)$ is hamiltonian.

An integer sequence $(a_1, a_2, \ldots, a_n)$ **such that** $0 \leq a_1 \leq a_2 \leq \ldots a_n < n$ **and** $n \geq 3$ **is hamiltonian if and only if the following holds for every** $i < n/2$**:** $a_i \leq i \rightarrow a_{n-i} \geq n - i$**.**

A graph G is called t-tough where $t > 0$ is any real number, if for every separator S, the graph $G - S$ has at most $|S|/t$ components. Hamiltonian graphs are clearly 1-tough.

Toghness Conjecture (Chvátal 1973): There exists an integer t such that every t-tough graph has a Hamilton Cycle.

The 4-color conjecture can be reduced to simple 3-connected maximal planar graphs, i.e to 3-connected triangulations. Considering the dual, we get that 4-color conjecture is equivalent to the assertion that every 3-connected cubic plane graph is 4-face colorable.

A 3-connected cubic plane graph is 4-face colorable if and only if it is 3-edge colorable. Thus 4-color conjecture is equivalent to the assertion that every 3-connected cubic graph is 3-edge colorable.

If every 3-connected cubic graph is hamiltonian the above assertion is true. Tutte demonstrated that this is not true.

Every 4-connected planar graph has a hamiltonian cycle.