Graph Theory: Lecture No. 31

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A network $N = N(x, y)$ is a digraph D (called the underlying digraph of N) with two distinct vertices x and y, called the source and sink respectively, together with a non-negative real valued function c defined on its edge set.

an $x - y$ flow (or simply a flow) in N is a real valued function defined on the edge set of N, satisfying the condition $f^+(v) = f^-(v)$ for all $v \in I$.
A feasible flow is one with the extra conditon that $0 \leq f(a) \leq c(a)$ for all edges a.

For any flow f in a network $N(x, y)$ and any subset X of V such that $x \in X$ and $y \in V \setminus X$, $\mathbf{val}(f) = f^+(X) - f^-(X)$

For any flow f and any cut K in a network N, $\mathbf{val}(f) \leq cap(K)$. Furthermore equality holds in this inequality if and only if each outgoing arc of X is f-saturated and each incoming arc of X is f-zero

Let f be a flow and K be a cut. If $\mathbf{val}(f) = cap(K)$ then f is a maximum flow and K is a minimum cut.

In any network, the value of a maximum flow is equal to the capacity of a minimum cut.