Graph Theory: Lecture No. 34

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Let $D = (V, A)$ be a directed graph. Suppose that with each arc a of D, are associated two real numbers $b(a)$ and $c(a)$ such that $b(a) \leq c(a)$. A circulation f in D is feasible (with respect to the functions b and c) if $b(a) \leq f(a) \leq c(a)$ for all $a \in A$. A feasible tension is defined analogously.

An obvious necessary condition for the existence of a feasible circulation: For each subset X **of** V**,** $c^+(X) \geq b^-(X)$

Hoffman's Circulation Theorem: A digraph D has a feasible circulation with respect to bounds b and c if and only if these bounds satisfy the above inequality for every subset X of V. Furthermore, if both b and c are integral valued, and satisfy this inequality, then D has an integer-valued feasible circulation.

Ghouila-Houri's Theorem: A digraph D has a feasible tension with respect to bounds b and c if and only if these bounds satisfy: $b(C^-) \leq c(C^+)$, for all cycles C in D with a sense of traversal. If b and c are integer valued and satisfy the above inequality, then D has an integer valued feasible tension.

A function f on the arc set A of digraph D is nowhere-zero if $f(a) \neq 0$ for each arc $a \in A$, i.e. if the support of f is the entire arc set A.

A nowhere zero circulation f **over** Z **in a digraph** D **is called a** k**-flow if** $-(k-1) \leq f(a) \leq (k-1)$**, for all** $a \in A$**.**

A graph admits a nowhere zero circulation over Z_k if and only if it admits a k-flow.

The flow number of a graph is defined to be the smallest positive integer k, for which it has a k-flow.

A graph admits a 2-flow if and only if it is even

A 2-edge connected cubic graph admits a 3-flow if and only if it is bipartite

Tutte's Flow conjectures:
(1) The 5**-flow conjecture: Every** 2**-edge connected graph admits a** 5**-flow.**
(2) The 4**-flow conjecture: Every** 2**-edge connected graph without a peterson graph minor admits a** 4**-flow.**
(3) The 3**-flow conjecture: Every** 2**-edge connected grpah without** 3**-edge cuts admits a** 3**-flow.**