$$\boxed{R(k) \le 2^{2k-3}}$$

$$2^{\frac{k}{2}} < R(k)$$

$$G \in \mathcal{G}\left(n, \tfrac{1}{2}\right) \qquad p = \tfrac{1}{2}$$

$$P\left(G \text{ has a } K_k \text{ in it}\right)$$

$$\leq \binom{n}{k} \cdot \frac{1}{2^{\frac{k(k-1)}{2}}}$$

$$= \frac{n(n-1)(n-2)\cdots(n-k+1)}{k!} \frac{1}{2^{\frac{k^2}{2} - \frac{k}{2}}}$$

$$\le \frac{n^k}{k!} \cdot \frac{1}{2^{\frac{k^2}{2}-\frac{k}{2}}}$$

$\underline{k!} \geq \underline{2^k}$ $k=4,5$

$\underline{1\cdot2\cdot3}\cdot4\cdot5\cdot \geq 2^5 = 32$ ✓

$120 \geq 32$

$24 \geq 2^4 = 16$ ✓

$k! \geq 2^k$

$\boxed{k > 3}$

$$\leq \frac{n^k}{2^k} \cdot \frac{1}{2^{\frac{k^2}{2} - \frac{k}{2}}}$$

$$n = 2^{\frac{k}{2}}$$

$$\leq \frac{\cancel{2^{\frac{k^2}{2}}}}{2^k} \frac{1}{\cancel{2^{\frac{k^2}{2}}} - \frac{k}{2}}$$

$$\leq \frac{1}{2^{k/2}} \leq \frac{1}{2} \checkmark$$

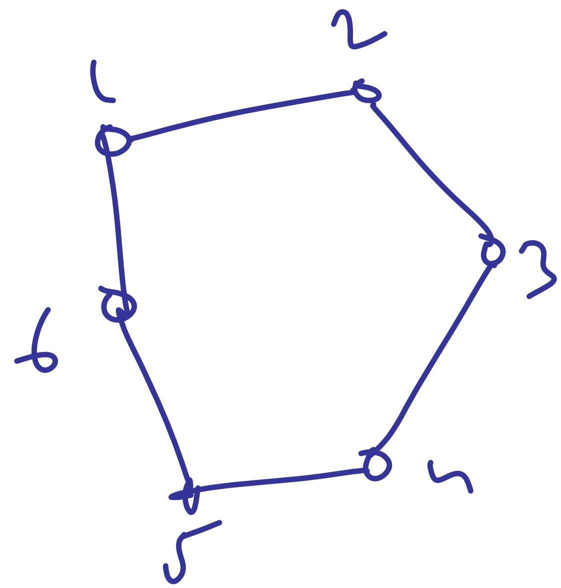
$$\Pr\left(G \text{ has a } K_k \text{ in it}\right) \leq \frac{1}{2}$$

$$\Pr\left(G \text{ has a } S_k \text{ in it}\right) < \frac{1}{2}$$

$$\Pr\left(G \text{ has either a } K_k \text{ or an } S_k \text{ in it}\right) < \frac{1}{2} + \frac{1}{2} = 1$$

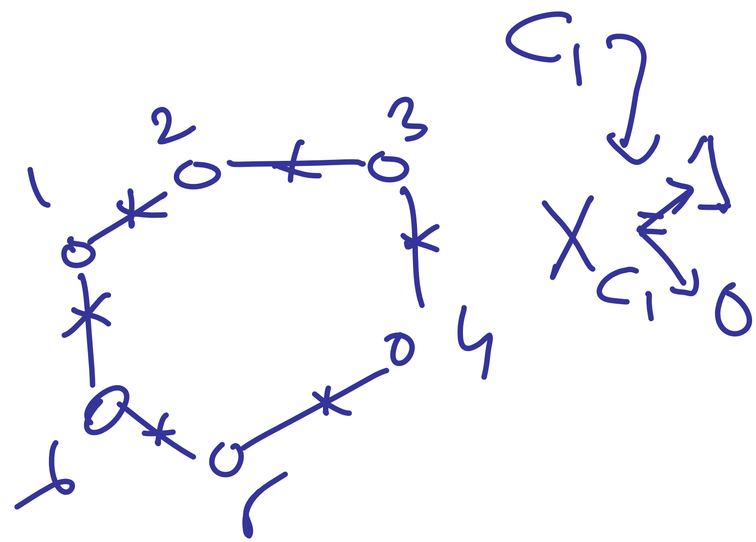
✓

$R(k) > 2^{k/2}$; $k \geq 3$


1
2
3
4
5
6

$\underline{\underline{X}}$ = # of k-cycles in the graph G

$\boxed{E(X)}$



k

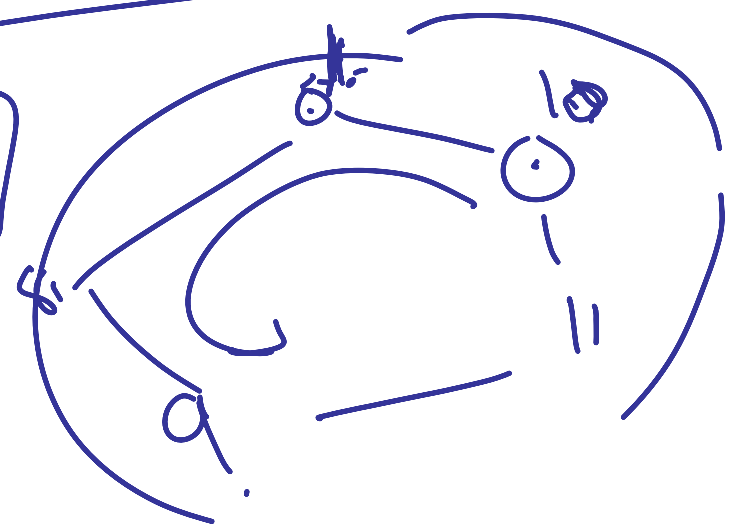
n

${}^{n}P_{k}$ ${}^{n}P_{k}$

$${}^{n}P_{k} = \boxed{n(n-1)(n-2)\cdots(n-(k+1))}$$

1, 10, 11, 9, 8

10, 11, 9, 8, 1



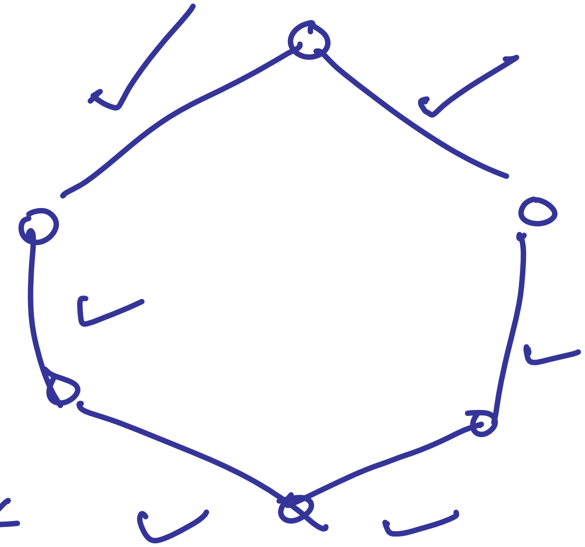
$$\frac{{}^{n}P_{k}}{2k} = \frac{n(n-1)\cdots(n-k+1)}{2k}$$

$$= \frac{(n)_k}{2k}$$

$$X_1,\ X_2,\ \cdots\cdots\ X_{\frac{(n)_k}{2k}}$$

$X_i = 1 \leftarrow p^k$

$X_i = 0 \quad 1 - p^k$



$$E(X_i) = p^k \cdot 1 + \cancel{(1-p^k)\cdot 0}$$

$$= p^k \quad \checkmark$$

$$E(X) = E\left(X_1 + X_2 + \cdots + X_{\frac{(n)_k}{2k}}\right)$$

$$= E(X_1) + E(X_2) + \cdots E\left(X_{\frac{(n)_k}{2k}}\right)$$

$$= \frac{(n)_k}{2k} \cdot p^k \quad \checkmark$$

$X \qquad Y$

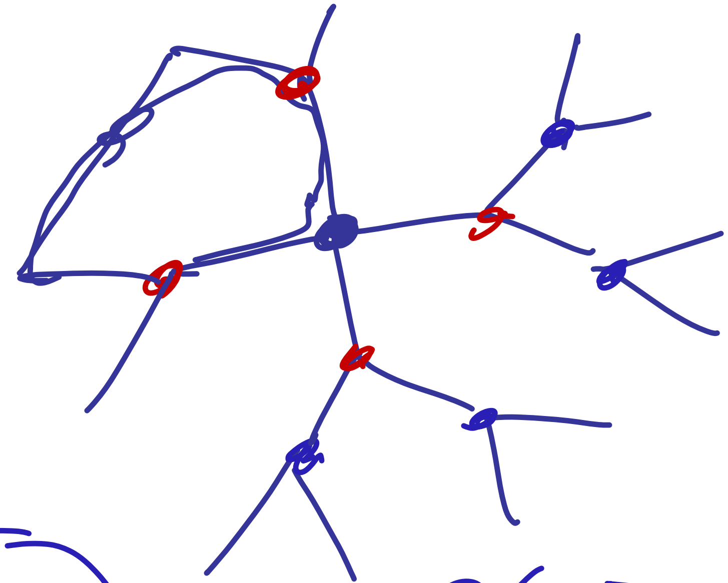
$$\boxed{E\left(\underbrace{aX + bY}\right) = \underline{aE(X) + bE(Y)}}$$

$$p(X > a) \leq \frac{E(X)}{a}$$

$$E(X) \geq \boxed{p(X > a)} \cdot a$$

$$\boxed{p(X > a) \leq \frac{E(X)}{a}}$$

$$\boxed{P(X > a) \le \frac{E(X)}{a}}$$



$$\boxed{\alpha \geq \frac{n}{2k}} \implies x \geq \frac{n}{\alpha} = \frac{n}{\left(\frac{n}{2k}\right)}$$

$$= 2k$$

$$\leq \binom{n}{t} q^{\frac{t(t-1)}{2}} \quad \Big\} \; \underline{\underline{q = 1-p}}$$

$$\leq \binom{n}{t} q^{\frac{t(t-1)}{2}}$$

$$\leq n^t q^{t\frac{(t-1)}{2}}$$

$$= \left(n\, q^{\frac{t-1}{2}}\right)^t$$

$$n\, q^{\frac{t-1}{2}} < 1$$

$$q = 1 - p \le e^{-p}$$

$$\boxed{1 - x \le e^{-x}}$$

$$q = 1 - p \le e^{-p}$$

$$\boxed{n\, e^{-p\frac{(t-1)}{2}}} = n\, n^{-3/2}\, e^{p/2} = \frac{\sqrt{e}}{\sqrt{n}} \to 0$$

$$p = \frac{6k \log n}{n} \qquad t = \frac{n}{2k} \qquad \underline{n \to \infty}$$