Graph Theory: Lecture No. 35

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Given an edge e let a probability space be associated with it, such that $\Omega_e = \{0_e, 1_e\}$ and choosing $P_e(1_e) = p$ and $P_e(0_e) = q = 1 - p$ as the probabilities of its two elementary events. We define the probability space $\mathcal{G}(n, p)$ by defining, $\Omega = \prod_{e \in [V]^2} \Omega_e$.
An element of Ω is a map ω assigning to each $e \in [V]^2$ either 0_e or 1_e and the probability measure P on Ω is the product measure of all the measures P_e.
We identify ω with the graph G on V such that $E(G) = \{e : \omega(e) = 1_e\}$, and call G a random graph on V with edge probability p.

Let $A_e = \{\omega : \omega(e) = 1_e\}$ **be the event consisting of all graph on** V **with** $e \in E(G)$**. Then the events** A_e **are independent and occur with probability** p**.**

For all integers n, k **with** $n \geq k \geq 2$**, the probability that** $G \in \mathcal{G}(n,p)$ **has a set of** k **independent vertices is at most**

$$P[\alpha(G) \geq k] \leq \binom{n}{k} q^{\binom{k}{2}}$$

For every $r \in N$, there exists $n \in N$ such that every graph of order at least n contains either K_r or $\overline{K_r}$ as an induced subgraph.

$$R(r) \leq 2^{2r-3}.$$

(Erd ös 1947) For every integer $k \geq 3$, the Ramsey Number of k satisfies: $R(k) > 2^{k/2}$.

For a random variable X on $\mathcal{G}(n, p)$, the mean or the expect value of X is given by:
$E(X) = \sum_{G \in \mathcal{G}(n,p)} P(G).X(G)$

Markov's Inequality: Let $X \geq 0$ be a random variable on $\mathcal{G}(n, p)$ and $a > 0$. Then:
$P[X \geq a] \leq \frac{E(X)}{a}$.