NPTEL Course on Numerical Optimization

Module 10 : Constrained Optimization - Algorithms

Practice Problems

1. Solve the following quadratic program and illustrate it geometrically.

$$\begin{array}{ll} \min & 2x_1^2 + x_2^2 + x_1x_2 - x_1 - x_2 \\ \text{s.t.} & x_1 + x_2 = 1 \end{array}$$

2. Solve the problem

$$\begin{array}{ll} \min & x_1^2 + x_2^2 - x_1x_2 - 3x_1 \\ \text{s.t.} & x_1 + x_2 \leq 4 \\ & x_1 \geq 0,\ x_2 \geq 0 \end{array}$$

   using the primal active set method starting at $(0, 0)^T$.

3. Use the primal active set method to solve the following quadratic programming problem:

$$\begin{array}{ll} \min & 4x_1^2 + x_2^2 + 2x_1x_2 + 2x_1 + 3x_2 \\ \text{s.t.} & x_1 - x_2 \geq 0 \\ & x_1 + x_2 \leq 4 \\ & x_1 \leq 3 \end{array}$$

   Choose three different starting points: one in the interior of the feasible region, one at a vertex and one at a non-vertex point on the boundary of the feasible region. Also verify the solution using graphical method.

4. Solve the following problem using penalty function method.

$$\begin{array}{ll} \min & x_1 + x_2 \\ \text{s.t.} & x_1^2 + x_2^2 = 2 \end{array}$$

   Draw the contours of $q(x, c^k)$ for $c^k = 1$ and $c^k = 10$.

5. Use augmented Lagrangian method to solve the following problem:

$$\begin{array}{ll} \min & x_1 + x_2 \\ \text{s.t.} & x_1^2 + x_2^2 = 2 \end{array}$$