NPTEL Course on Numerical Optimization Module 3 : Unconstrained Optimization (One Dimensional Optimization) Practice Problems

1. Consider the following problems:

- (a) Minimize x^3 subject to $0 < x \le 1$
- (b) Minimize $\log x$ subject to $x \ge 1$

For every problem, check if an optimal solution exists using graphical method. Justify your answer.

- 2. The function $f(x) = -5x^5 + 4x^4 12x^3 + 11x^2 2x + 1$ is unimodal in the interval [-0.5, 0.5]. Use dichotomous search, Fibonacci search and golden-section search to find the minimizer of f with the length of final interval of uncertainty less that 10^{-5} .
- 3. Find the maximum value of $f(x) = 2 \sin x \frac{x^2}{10}$ using Newton method (with $x^0 = 2.5$). Also solve this problem using Golden Section search with the initial interval of uncertainty, [0, 4].
- 4. Apply Newton's method to find a minimum of the function

$$f(x) = \frac{11}{546}x^6 - \frac{38}{364}x^4 + \frac{1}{2}x^2$$

with $x^0 = 1.01$. Verify that f^k is a monotonically decreasing sequence. Does $\{x^k\}$ converge to a local minimum?

5. Prove the following inequality for all x > 0 using optimality conditions:

$$\frac{1}{x} + x \ge 2.$$