NPTEL Course on Numerical Optimization

Module 4 : Convex Sets

Practice Problems

1. Which of the following sets are convex?

   - $\{\mathbf{x} \in \mathbb{R}^n : \|\mathbf{x}\|_2^2 \geq 1\}$
   - $\{\mathbf{x} \in \mathbb{R}^n : \|\mathbf{x}\|_2^2 = 1\}$
   - $\{\mathbf{x} \in \mathbb{R}^n : \max_{i=1,\ldots,n} x_i \geq 1\}$
   - $\{\mathbf{x} \in \mathbb{R}^n : \min_{i=1,\ldots,n} x_i \leq 1\}$

2. Let $C_1 = \{(x, y) : x^2 + y^2 = 1\}$ and $C_2 = \{(x, y) : (x-2)^2 + y^2 \leq 1\}$.

   - Show that C_1 and C_2 are closed convex subsets of $\mathbb{R}^2$.
   - Are C_1 and C_2 strictly separable? Are C_1 and C_2 strongly separable?
   - Draw the convex hull of $C_1 \cup C2$.

3. Is it true that the union of two convex sets is a convex set? If not, find a counterexample.

4. Let $S_1 = \{(x, y) : y \leq 0\}$ and $S_2 = \{(x, y) : y \geq \frac{1}{x}, x \neq 0\}$. Find a hyperplane which separates S_1 and S_2. Is this a strictly separating hyperplane?

5. Find all the supporting hyperplanes of the set $C = \{(x, y) : x \geq 0\} \cap \{(x, y) : y \geq 0\} \cap \{(x, y) : x^2 + y^2 = 1\}$.