NPTEL Course on Numerical Optimization

Module 5 : Convex Functions

Practice Problems

1. Discuss the convexity and concavity of the following functions:

   (a) $f(\mathbf{x}) = \log(\sum_{i=1}^{n} e^{x_i})$

   (b) $f(\mathbf{x}) = e^{\mathbf{x}^T \mathbf{A} \mathbf{x}}$ where $\mathbf{A}$ is a positive definite matrix.

   (c) $f(\mathbf{x}) = \|\mathbf{x}\|_2$

   (d) $f(\mathbf{x}) = -x_1^2 - 4x_2^2 - 9x_3^2 + 2x_1x_2 + 3x_1x_3 + 6x_2x_3$

   (e) $f(x) = \sqrt{1 - x^2}, -1 \leq x \leq 1$

   (f) $f(\mathbf{x}) = \log(x_1^a x_2^a \dots x_a^n)$ where $x_i > 0 \ \forall\ i$ and $a > 0$

2. Prove that $2e^{x+y} \leq e^{2x} + e^{2y}$ for all $x, y \in \mathbb{R}$.

3. Are the following problems convex programming problems? Justify your answer.

   (1)

$$\begin{array}{ll} \max & \log(1 + x_1) + x_2 \\ \text{s.t.} & 2x_1 + x_2 \leq 3 \\ & x_1, x_2 \geq 0 \end{array}$$

   (2)

$$\begin{array}{ll} \min & |x - 1| + |x - 4| \\ \text{s.t.} & 0 \leq x \leq 5 \end{array}$$

4. Solve the problem:

$$\begin{array}{ll} \min & (x_1 + 2x_2 - 3)^2 \\ \text{s.t.} & x_1, x_2 \in \mathbb{R} \end{array}$$

5. If f and h are convex functions, then show that the function $\max\{f, h\}$ is also convex.

6. Show that a convex function defined on a closed real interval attains its maximum at one of the endpoints of the interval.