NPTEL Course on Numerical Optimization

Module 6 : Unconstrained Optimization

(Multi-dimensional Optimization)

Practice Problems

1. Investigate the stationary points of the following functions:

   (a) $f(\boldsymbol{x}) = x_1^2x_2^2 - 4x_1^2x_2 + 4x_1^2 + 2x_1x_2^2 + x_2^2 - 8x_1x_2 + 8x_1 - 4x_2$

   (b) $f(\boldsymbol{x}) = 2x_1^3 - 3x_1^2 - 6x_1x_2(x_1 - x_2 - 1)$

   (c) $f(\boldsymbol{x}) = 2x_1^2 + x_2^2 - 2x_1x_2 + 2x_1^3 + x_1^4$

2. Let $f(\boldsymbol{x}) = (x_1 + x_2^2)^2$. Verify that $d = (-1, 1)^T$ is a descent direction of f at the point $(1, 0)^T$. Use exact line search method to determine all permissible step lengths.

3. Consider the problem,

   $$\min \quad 10x_1^2 + x_2^2$$

   Apply the following methods to solve this problem, using $\boldsymbol{x}^0 = (1/10, 1)^T$.

   (a) Steepest descent method with exact line search

   (b) Classical Newton Algorithm

   (c) DFP method with exact line search and $\mathbf{B}^0 = \boldsymbol{I}$

   (d) Conjugate Gradient method of Fletcher-Reeves with exact line search

   Sketch the contours of the function and also the sequences generated by each algorithm. Compare the directions obtained using DFP method and Conjugate Gradient method and also verify that $\mathbf{B}^n = \boldsymbol{H}^{-1}$.

4. Consider the problem to minimize $f(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^T\boldsymbol{H}\boldsymbol{x} + \boldsymbol{c}^T\boldsymbol{x}$ where $\boldsymbol{H}$ is a symmetric positive definite matrix. If DFP method is used with $\mathbf{B}^0 = \boldsymbol{I}$, then show that

   - ${\boldsymbol{d}^i}^T\boldsymbol{H}\boldsymbol{d}^j = 0 \; \forall \; i \neq j$
   - $\mathbf{B}^{k+1}\boldsymbol{H}\boldsymbol{d}^i = \boldsymbol{d}^i \; \forall \; i = 0 \rightarrow k$

5. Use the coordinate descent method with exact line search to minimize

$$f(\boldsymbol{x}) = 4x_1^2 + x_2^2 - 2x_1x_2$$

and compare the iterates with those given by Conjugate Gradient (Fletcher-Reeves) method with exact line search. Sketch the contours of the function and also the sequences $\{x^k\}$ generated by the two methods. Use the same $\boldsymbol{x}^0$ for both the methods.