NPTEL Course on Numerical Optimization

Module 7 : Constrained Optimization

Practice Problems

1. Consider the problem of finding the point on the parabola $x_2 = \frac{1}{5}(x_1 - 1)^2$ that is closest, in the Euclidean norm sense, to $(1, 2)^T$.

   (a) Use KKT conditions to solve this problem.

   (b) By eliminating the variable x_1, convert this problem into an unconstrained optimization problem and solve it.

2. Find the rectangle of a given perimeter that has greatest area by solving the first order necessary conditions. Verify that the second order sufficiency conditions are satisfied.

3. Consider the constraint set, $\{(x_1, x_2) \in \mathbb{R}^2 : x_2 - (x_1 - 1)^2 \leq 0, x_1 \geq 0, x_2 \geq 0\}$. Is the point $(1, 0)^T$ feasible and regular?

4. Maximize $14x_1 - x_1^2 + 6x_2 - x_2^2 + 7$ subject to $x_1 + x_2 \leq 2$ and $x_1 + 2x_2 \leq 3$.

5. Minimize $x_1^2 + x_2^2 + \frac{1}{4}x_3^2$ subject to $x_1 - x_3 + 1 = 0$ and $x_1^2 + x_2^2 = 2x_1$.