NPTEL Course on Numerical Optimization

Module 9 : Linear Programming

Practice Problems

1. Solve the following linear program using graphical method:

$$
\begin{aligned}
\max \quad & -x_1 + x_2 \\
\text{s.t.} \quad & x_1 - x_2 \leq 2 \\
& x_1 + x_2 \leq 6 \\
& x_1 \geq 0, x_2 \geq 0
\end{aligned}
$$

2. Convert the following linear program to standard form:

$$
\begin{aligned}
\max_{\boldsymbol{x},\boldsymbol{y}} \quad & \boldsymbol{c}^T\boldsymbol{x} + \boldsymbol{d}^T\boldsymbol{y} \\
\text{s.t.} \quad & \mathbf{A}_1\boldsymbol{x} = \mathbf{b}_1 \\
& \mathbf{A}_2\boldsymbol{x} + \mathbf{B}_2\boldsymbol{y} \leq \mathbf{b}_2 \\
& l \leq y_i \leq u \; \forall \, i
\end{aligned}
$$

3. Convert the following problem to a linear programming problem:

$$
\begin{aligned}
\min \quad & f(\boldsymbol{x}) \\
\text{s.t.} \quad & \mathbf{A}\boldsymbol{x} = \mathbf{b} \\
& \boldsymbol{x} \geq 0
\end{aligned}
$$

where $f(\boldsymbol{x}) = \max(\boldsymbol{c}_1^T\boldsymbol{x} + d_1, \boldsymbol{c}_2^T\boldsymbol{x} + d_2, \ldots, \boldsymbol{c}_p^T\boldsymbol{x} + d_p)$.

4. Using the simplex method solve

$$
\begin{aligned}
\min \quad & 2x_1 - x_2 \\
\text{s.t.} \quad & 2x_1 - x_2 - x_3 \geq 3 \\
& x_1 - x_2 + x_3 \geq 2 \\
& x_i \geq 0 \; \forall \, i
\end{aligned}
$$

What is the dual problem and its solution?

5. Solve the following problem:

$$
\begin{aligned}
\min \quad & 3x_1 - 5x_2 + x_3 \\
\text{s.t.} \quad & x_1 - 2x_3 \geq 4 \\
& 2x_1 - x_2 + x_3 \geq 2 \\
& x_i \geq 0 \; \forall \, i
\end{aligned}
$$