Power system Dynamics and Control

General Concepts of Control and Dynamics

1. The discrete time system given by

$$x_{k+1} = -1.5x_k$$

is:
(a) Stable (b) Unstable (c) Marginally-stable (d) Cannot Say

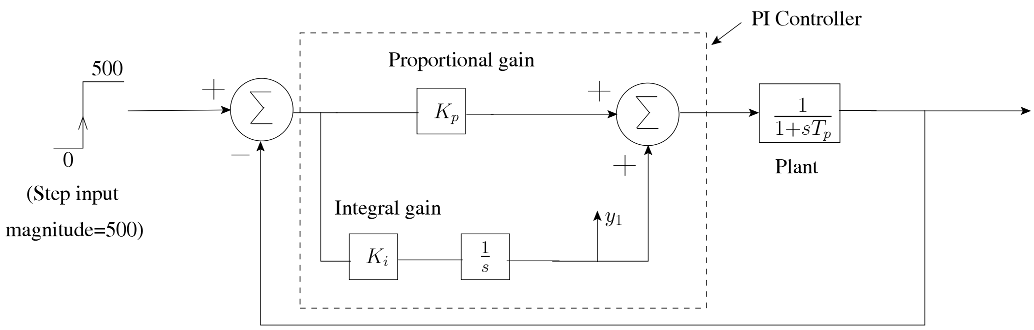
2. Consider the following equation :

$$\frac{dx_1}{dt} = x_2$$
$$\frac{dx_2}{dt} = -x_1$$

If $\theta = \tan^{-1}\frac{x_2}{x_1}$ and $W = \frac{1}{2}(x_1^2 + x_2^2)$, then write down the expression for :

$$\frac{dW}{dt} =$$
$$\frac{d\theta}{dt} =$$

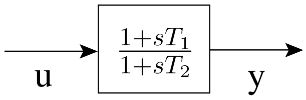
3. If the control system shown in the figure below is stable, and $K_p \neq 0$, $K_i \neq 0$,the value of y_1, in the steady state is:

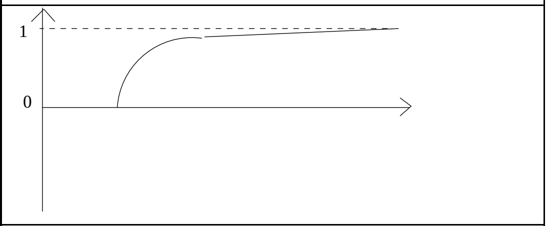
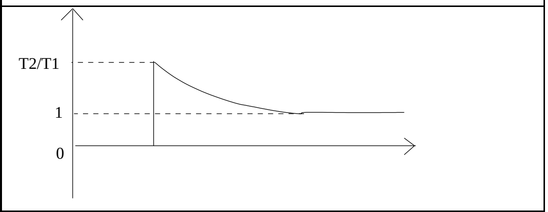
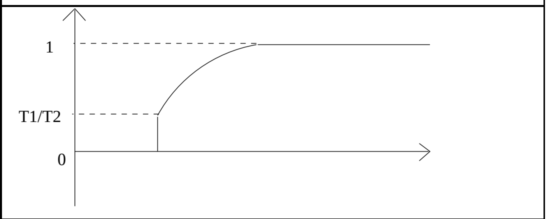
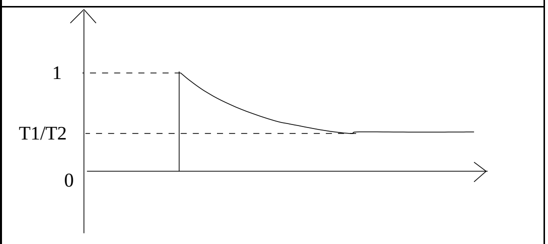


(a) 500 (b) Zero (c) $\frac{500}{K_p}$ (d) 500 K_i

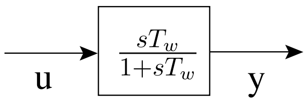
4. $\int_0^t e^{-\lambda\tau}\, d\tau =$
(a) $\frac{e^{-\lambda t}-1}{\lambda}$ (b) $\frac{1-e^{-\lambda t}}{\lambda}$ (c) $-\lambda e^{-\lambda t}$ (d) $\frac{-e^{-\lambda t}}{\lambda}$

5. The response of the system shown below for a unit step input is ($T_1 < T_2$):



	Response
A)	
B)	
C)	
D)	

6. A state space representation of the following system is :



(a) $\dot{x} = -\frac{1}{T_w}x + \frac{1}{T_w}u$ and $y = x$ (b) $\dot{x} = -\frac{1}{T_w}x + \frac{1}{T_w}u$ and $y = \frac{1}{T_w}x$
(c) $\dot{x} = -\frac{1}{T_w}x + \frac{1}{T_w}u$ and $y = u - x$ (d) $\dot{x} = -\frac{1}{T_w}x + \frac{1}{T_w}u$ and $y = u + x$

7. If for the above system, u is a unit step input, the steady state value of y will be :

8. Consider the linear system :

$$\dot{x} = Ax + bu$$

$$y = Cx + du$$

and

$$\dot{z} = R^{-1}ARz + R^{-1}bu$$
$$y = CRz + du$$

Which of the following statement(s) is/are true:
(a) The transfer functions $\frac{y(s)}{u(s)}$ for both systems are identical
(b) The eigenvalues of A and $R^{-1}AR$ are identical
(c) The eigenvectors of A and $R^{-1}AR$ are identical
(d) If the first system is stable so is the second

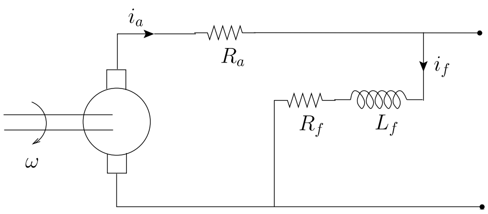
9. Consider the system given below:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} \lambda_1 & \mu_1 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Answer the following question assuming that $\mu_1 \neq 0$
(a) What are eigenvalues corresponding to 2 modes ?
(b) Are both modes observable in the state x_1 ? if no, which is not ?
(c) Are both modes observable in the state x_2 ? if no, which is not ?
(d) Are both modes controllable by input u ? if no, which is not ?
If $\mu_1 = 0$, answer the above question again.

10. If input u in the above question is zero, can you find a set of initial condition of the states for which the mode corresponding to the eigenvalue λ_1 is not excited (i.e. the time response of both x_1 and x_2 does not have the term $e^{-\lambda_1 t}$). Also find the initial conditions if the other modes are not to be excited.
If both modes are not to be excited, what should be the initial condition ?

11. Consider the equations of an self excited DC generator which is driven by a constant speed prime mover running at a speed $= \omega$. The generator is operating at no load, therefore $i_a = i_f$.



$$\frac{d\psi}{dt} = k\psi\omega - (R_a + R_f)i_f$$
$$\psi = g(i_f)$$

ψ is the field flux - neglect the effect of armature reaction, k is a constant depending on the parameters of the machine (number of turns etc). 'g' denotes the nonlinear relationship between the field flux (ψ) and the field current i_f. Note that the function 'g' has the characteristics of a typical saturation function: also when $i_f = 0$, then $\psi = 0$.
Analytically derive the conditions for self excitation.

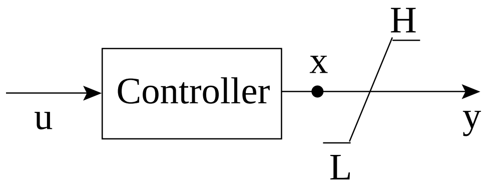
12. Consider the dynamical system

$$\dot{x} = -x^2$$

The equilibrium point of the system is x= ....
If this system is 'linearized' around the equilibrium point, what does the model predict as far as the small disturbance behavior is concerned ? Is it a correct prediction ?

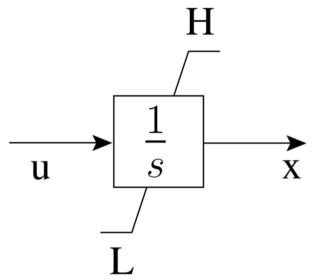
13. The output of most controllers used in a power system are limited in some way (why?). This may be achieved by simply saturating the output when it exceeds a certain value. This is called a : 'soft limiter' . The governing equations are:

$$y = x \text{ if } L \leq x \leq H.$$
$$y = L \text{ if } L > x.$$
$$y = H \text{ if } H < x.$$

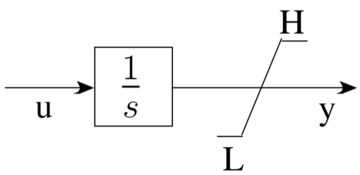


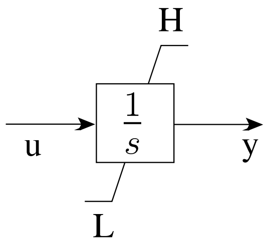
Another type of limiter is a 'hard limiter' which is applied to integrator blocks in a controller. The governing equations are :

$$\frac{dx}{dt} = u \text{ if } L \leq x \leq H.$$
$$\frac{dx}{dt} = 0 \text{ if } x \leq L \text{ and } u < 0.$$
$$\frac{dx}{dt} = 0 \text{ if } x \geq H \text{ and } u > 0.$$



Suppose the following systems are fed an input $u = \sin t$ and $H = 0.5$, $L = -0.5$. Plot the response of the output in each case.





14. Draw a block diagram which implements the following control function- input u, output y:
y changes at a rate given by $20(u_{max} - u)$ if $u \leq u_{max}$
y changes at a rate given by $2000(u_{max} - u)$ if $u_{max} < u$
y : maximum positive rate of change of $y = +0.05/s$
y : negative rate of change of y - no limit
y : hard limit: maximum value : 0.02 pu, minimum value 0
You may use blocks like integrators, compensator or summers and limiters.

15. Consider a system $\dot{x} = Ax$, for a small disturbance, consider its response (i.e. match the columns).

(a) A= $\begin{bmatrix} -10 & 0.1 \\ 0.1 & -15 \end{bmatrix}$

(b) A= $\begin{bmatrix} 0 & 1 \\ -55 & -0.1 \end{bmatrix}$

(c) A= $\begin{bmatrix} 10 & 0.1 \\ 0.1 & 15 \end{bmatrix}$

(d) A= $\begin{bmatrix} 0 & 55 \\ -55 & 0 \end{bmatrix}$

(e) A= $\begin{bmatrix} 0.1 & 55 \\ -55 & 0 \end{bmatrix}$

(a) Damped Oscillation (decaying)

(b) Undamped Oscillation (sustained)

(c) Growing oscillation

(d) Exponential decaying (monotonically) response

(e) Exponential growing (monotonically) response

16. For a system $\dot{x} = Ax$, where A= $\begin{bmatrix} -5 & 0 \\ 0 & 6 \end{bmatrix}$, give an initial condition '$x(0)$' which will excite only the stable mode.

17. For a system $\dot{x} = Ax$, where A= $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, obtain the eigenvalues and eigenvectors
The modes of this system may be described as a "common" mode and one "differential" mode, why ?

18. For a system A= $\begin{bmatrix} 1 & 2 & 3 \\ 40 & 5 & 6 \\ 7 & 50 & 9 \end{bmatrix}$ Give a set of initial conditions such that the unstable mode is NOT excited.

19. For the system

$$\dot{x} = \begin{bmatrix} 5 & 1 \\ 0 & -2 \end{bmatrix} x$$

for some initial condition of x only the stable mode is excited. Give any such initial condition.

20. Consider the transformation $x = Py$ where

$$P = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix}$$

(a) Shown that $P^{-1} = P^T$

(b) Hence show that if $x^T = [x1 \;\; x2]$ and $y^T = [y1 \;\; y2]$, then $|\, x \,| = \sqrt{x_1^2 + x_2^2} = |\, y \,| = \sqrt{y_1^2 + y_2^2}$

(c) What are the eigenvalues of P ?

(d) If A is a real symmetric matrix show that one can find α such that $P^{-1}AP$ is a diagonal matrix.

(e) Comment on the above result.

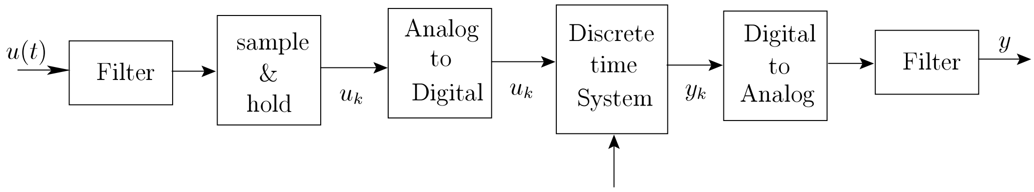
21. (a) Give a state space representation of the transfer function

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

(b) Comment on the low and high frequency gain of the transfer function.

(c) If $\omega_n = 300\ rad/s$ and $\zeta = 0.1$, what would you call "high" and "low" frequency in the above question ?

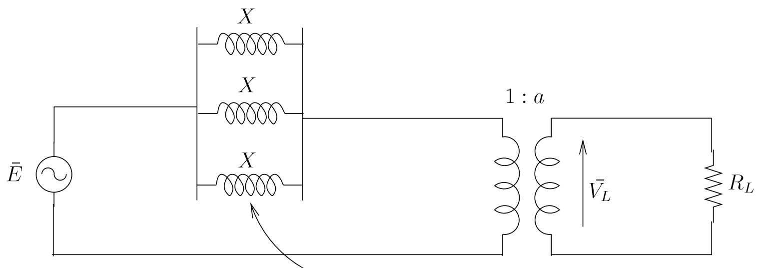
(d) Suppose the above transfer function, with $\omega_n = 300\ rad/s$ and $\zeta = 0.1$, is to be implemented numerically as shown below, write down the discrete-time equations(algorithm) for the same (with appropriate explanation).



(e) How will you choose the sampling time ?

(f) What is the role of the filters shown in the diagram above ?

22. Consider the system given below. A resistive load of ($R_L = 1$ pu) is being supplied via a tap-changing transformer which attempts to maintain the magnitude of voltage at the load, V_L, at 1 pu. For simplicity of analysis assume that the tap can be varied in a vernier



fashion (in actual practice, only discrete variation in tap is possible). In order to regulate V_L at 1.0 pu, the tap is varied by an integral control action as follows:

$$\dot{a} = k(1.0 - V_L)$$

Note: $k = 5.0$. V_L is a function[1] of a. The maximum tap value is 1.5 and minimum value is 0.75.

[1]For analysis of the tap changing action you may assume that the network is always in sinusoidal steady state - this is justifiable since the tap changing actions are very slow compared to the natural transients of the network. Therefore you may use phasor analysis to obtain the magnitude of the voltage V_L

(a) Suppose the system is in equilibrium such that $a = 1$. Compute the magnitude of voltage E if $X = 1.5$pu.

(b) Is the system stable for small disturbance around this equilibrium ?

(c) Now due to a disturbance one of the line trips. What is the new equilibrium value of a and V_L ? E is kept at its pre-disturbance value.

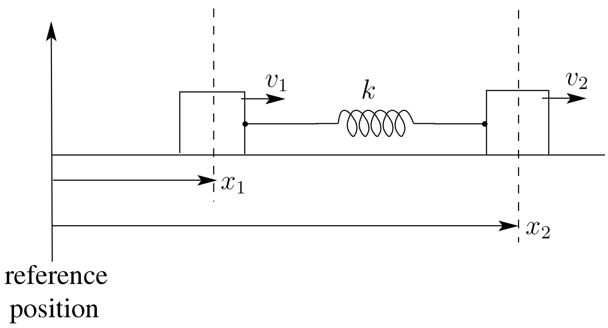
(d) Is the system stable at this new equilibrium ?

(e) If due to an additional disturbance one more line trips. What is the new equilibrium value of a and V_L ? E is kept at its pre-disturbance value.

(f) If the tap was 'fixed' at the value prior to the last disturbance and not varied, what would be the equilibrium value of V_L.

(g) Comment on your results

23. Consider a 2 mass-spring system. The dynamic equations of the system are



x_1, x_2 : position of the 2 masses
v_1, v_2 : speed of the 2 masses
M_1, M_2 : Mass
k : spring constant

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ M_1\dot{v_1} \\ M_2\dot{v_2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k & k & 0 & 0 \\ k & -k & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ v_1 \\ v_2 \end{bmatrix}$$

(a) Obtain the characteristic equation and show that there is a pair of zero and a conjugate pair of imaginary eigenvalues. [2]

(b) Show that the mode corresponding to the zero eigenvalues is not observable in the signals ($y_1 = x_1 - x_2$) and ($y_2 = v_1 - v_2$)

(c) Show that the oscillatory mode is not observable in the signals $y_3 = \dfrac{M_1x_1 + M_2x_2}{M_1 + M_2}$ and $y_4 = \dfrac{M_1v_1 + M_2v_2}{M_1 + M_2}$

24. Find out the time response of the system of equations $\dot{x} = Ax$ and plot the response for $x(t)$ where $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ and initial conditions are :

Case I: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ Case II: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

[2] $k > 0, M_1 > 0$ and $M_2 > 0$

Case III: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} p11 \\ p21 \end{bmatrix}$ Case IV: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} p12 \\ p22 \end{bmatrix}$

where $P = \begin{bmatrix} p11 & p12 \\ p21 & p22 \end{bmatrix}$ is the transformation matrix which makes $P^{-1}AP$ diagonal.

25. Find out the time response of the system of equations $\dot{x} = Ax$ where $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and initial conditions are : $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

26. Find out the time response of the system of equations $\dot{x} = Ax$ where $A = \begin{bmatrix} 0 & 1 \\ -1 & -0.1 \end{bmatrix}$ and initial conditions are : $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

27. Find out the time response of the system of equations $\dot{x} = Ax$ where $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ and initial conditions are : $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$