

Power system Dynamics and Control  
**Numerical Techniques**

1. Write down the state space equations for the linear circuit shown in Fig. 1.

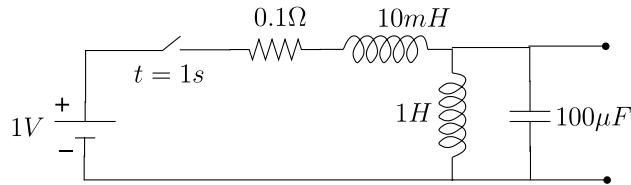


Figure 1

2. Evaluate the eigenvalues of the A matrix of the system. Write down the eigenvectors as well (in case of complex numbers, give the magnitude and angle).
3. Repeat the same exercise for the circuit shown in Fig. 2.

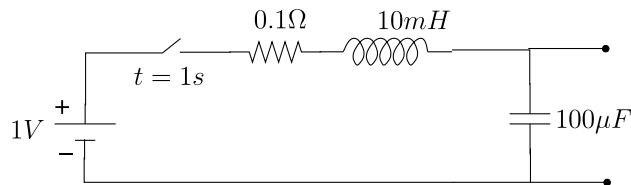


Figure 2

4. Repeat the same exercise for the circuit shown in Fig. 3.

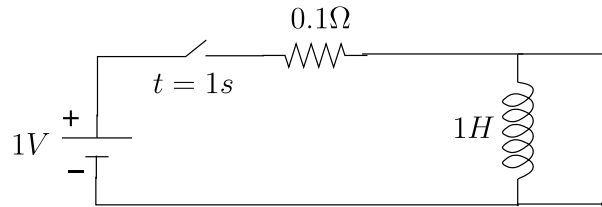
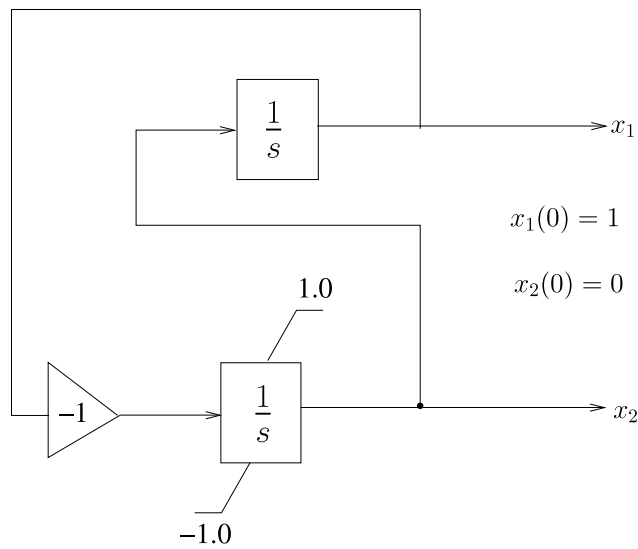


Figure 3

5. Comment on your results.
6. Assuming zero initial conditions, simulate all the circuits using an explicit method like Forward Euler. Comment on your result and also on the performance of the numerical method (choice of time step etc.)
  - (a) Plot all the states versus time on a scale of 0-30 s.
  - (b) Plot all the states versus time on a scale of 1-1.2 s.
7. Repeat the same exercise using Trapezoidal and Backward Euler Rule of integration.

8. Simulate the following system using Euler Method. Comment on your result.



9. Suppose you intend to simulate a several systems with the following characteristics. Match the numerical method which you should use for the simulation. Note that more than one match are possible for each. Time step of simulation is denoted by  $h$ .

**Note:** You are free to give alternative answers not listed below

	System To be Simulated		Appropriate Method
1	Stiff System: Only slow dynamics are of interest	A	Backward Euler with $h$ compatible with the time period (for oscillatory behaviour) / time constant (for exponential monotonic behaviour) associated with slow dynamics
2	Stiff System : Both slow and fast dynamics are to be captured accurately	B	Backward Euler or Trapezoidal Rule with $h$ compatible with the time period (for oscillatory behaviour)/ time constant (for exponential monotonic behaviour) associated with the slow dynamics
3	Non-Stiff System	C	Trapezoidal Rule with $h$ compatible with the time period (for oscillatory behaviour)/ time constant(for exponential monotonic behaviour), associated with fast dynamics
4	Stiff System : Only interested in fast dynamics	D	Trapezoidal Rule with $h$ compatible with the time period(for oscillatory behaviour)/ time constant(for exponential monotonic behaviour).
5	Nonlinear System, time required for computation <i>in each step</i> has to be guaranteed to be within a certain period	E	Fourth order Runge Kutta method with $h$ compatible with the time period(for oscillatory behaviour)/ time constant(for exponential monotonic behaviour) associated with fast dynamics
6	System with extremely low damped oscillatory behaviour - the damping (positive or negative) has to faithfully replicated	F	Forward Euler method with $h$ compatible with the time period(for oscillatory behaviour)/ time constant(for exponential monotonic behaviour) associated with slow dynamics
		G	Backward Euler with variable $h$ , initially small to capture fast dynamics and then later made larger (after the fast dynamics have decayed) for improved speed.
		I	Forward Euler with variable $h$ , initially small to capture fast dynamics and then later made larger (after the fast dynamics have decayed) for improved speed.

10. Consider the following stiff system:

$$\begin{aligned}\dot{x}_s &= f_1(x_s, x_f) \\ \dot{x}_f &= f_2(x_s, x_f)\end{aligned}$$

where  $x_s$  and  $x_f$  are mainly ‘associated’ with the slow and fast transients respectively. Answer the following questions.

- (i) If *only* slow transients are of interest, and it is known that the fast transient is stable, then which of the following methods is suitable for simulation.

(A) Use the simplified model:

$$\begin{aligned}\dot{x}_s &= f_1(x_s, x_f) \\ 0 &= f_2(x_s, x_f)\end{aligned}$$

and then simulate by an implicit or explicit method with a fixed time step compatible with the slow transient.

(B) Use the simplified model:

$$\begin{aligned}0 &= f_1(x_s, x_f) \\ \dot{x}_f &= f_2(x_s, x_f)\end{aligned}$$

and then simulate by implicit or explicit method like Runge-Kutta method with a fixed time step compatible with the slow transient.

(C) Simulate using explicit method like Runge-Kutta method with a fixed time step compatible with the slow transient, without modelling simplifications

(D) Simulate using implicit method like Backward Euler Method, with a fixed time step compatible with the slow transient, with/without modelling simplifications

- (ii) For the previous question, if one uses the correct modeling approximation, will it give correct results in the steady state ?

11. For the stiff system given in the previous questions, if one wants to accurately obtain the transient response for a very short time duration (immediately after a disturbance), then which of the following approximations would be appropriate ?

(A) Use the simplified model:

$$\begin{aligned}\dot{x}_s &= f_1(x_s, x_f) \\ 0 &= f_2(x_s, x_f)\end{aligned}$$

(B) Use the simplified model:

$$\begin{aligned}0 &= f_1(x_s, x_f) \\ \dot{x}_f &= f_2(x_s, x_f)\end{aligned}$$

(C) Use the simplified model:

$$\begin{aligned}\dot{x}_s &= 0, \\ \dot{x}_f &= f_2(x_s, x_f)\end{aligned}$$

(D) Use the simplified model:

$$\begin{aligned} \dot{x}_s &= f_1(x_s, x_f) \\ \dot{x}_f &= 0 \end{aligned}$$

12. For the previous question, will the approximation give correct results in the steady state?
13. To simulate a stiff system, wherein *both* fast and slow transients are of interest, while maintaining speed and accuracy, one should use :
- (a) A fixed time step explicit numerical integration method like R-K
  - (b) A variable time step explicit numerical integration method like R-K
  - (c) A fixed step time implicit numerical integration method like Trapezoidal Rule
  - (d) A variable time step implicit numerical integration method like Trapezoidal Rule