Power system Dynamics and Control
Synchronous Machine Modelling

1. Consider the following transformation:

$$\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} = C_{P_1} \begin{bmatrix} f_{d1} \\ f_{q1} \\ f_{01} \end{bmatrix} \quad , \quad \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} = C_{P_2} \begin{bmatrix} f_{d2} \\ f_{q2} \\ f_{02} \end{bmatrix}$$

   where

$$C_{P_1} = \sqrt{\tfrac{2}{3}} \begin{bmatrix} \cos(\phi_1) & \sin(\phi_1) & \frac{1}{\sqrt{2}} \\ \cos(\phi_1 - \frac{2\pi}{3}) & \sin(\phi_1 - \frac{2\pi}{3}) & \frac{1}{\sqrt{2}} \\ \cos(\phi_1 - \frac{4\pi}{3}) & \sin(\phi_1 - \frac{4\pi}{3}) & \frac{1}{\sqrt{2}} \end{bmatrix}$$

   and

$$C_{P_2} = \sqrt{\tfrac{2}{3}} \begin{bmatrix} \cos(\phi_2) & \sin(\phi_2) & \frac{1}{\sqrt{2}} \\ \cos(\phi_2 - \frac{2\pi}{3}) & \sin(\phi_2 - \frac{2\pi}{3}) & \frac{1}{\sqrt{2}} \\ \cos(\phi_2 - \frac{4\pi}{3}) & \sin(\phi_2 - \frac{4\pi}{3}) & \frac{1}{\sqrt{2}} \end{bmatrix}$$

   (a) Obtain the relationship between $\begin{bmatrix} f_{d1} \\ f_{q1} \\ f_{01} \end{bmatrix}$ and $\begin{bmatrix} f_{d2} \\ f_{q2} \\ f_{02} \end{bmatrix}$

   i.e if $\begin{bmatrix} f_{d2} \\ f_{q2} \\ f_{02} \end{bmatrix} = P \begin{bmatrix} f_{d1} \\ f_{q1} \\ f_{01} \end{bmatrix}$ then find [P]

   (b) Show that $(f_{q2} + jf_{d2}) = (f_{q1} + jf_{d1})e^{j(\phi_1 - \phi_2)}$ and hence show that $\sqrt{f_{q2}^2 + f_{d2}^2} = \sqrt{f_{q1}^2 + f_{d1}^2}$

   Assume in the following problems that :

$$F^{abc} = C(\gamma)F^{dq0} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\gamma) & \sin(\gamma) & \frac{1}{\sqrt{2}} \\ \cos(\gamma - \frac{2\pi}{3}) & \sin(\gamma - \frac{2\pi}{3}) & \frac{1}{\sqrt{2}} \\ \cos(\gamma - \frac{4\pi}{3}) & \sin(\gamma - \frac{4\pi}{3}) & \frac{1}{\sqrt{2}} \end{bmatrix} F^{dq0} \tag{1}$$

2. If the abc variables are transformed by two transformations $C(\gamma_1)$ and $C(\gamma_2)$ to two sets of variables, $(d_1, q_1, 0)$ and $(d_2, q_2, 0)$, where $\gamma_1 = \gamma_2 + \phi$, then if $v_0 = 0$:

   (a) Instantaneous Real Power $= v_{q1}i_{q1} + v_{d1}i_{d1} = v_{q2}i_{q2} + v_{d2}i_{d2}$
   (b) Instantaneous Real Power $= v_{q1}i_{q1} + v_{d1}i_{d1} = (v_{q2}i_{q2} + v_{d2}i_{d2})e^{j\phi}$
   (c) Instantaneous Real Power $= v_{q1}i_{q1} + v_{d1}i_{d1} = (v_{q2}i_{q2} + v_{d2}i_{d2})e^{-j\phi}$
   (d) Instantaneous Real Power $= v_{q1}i_{q1} + v_{d1}i_{d1} = (v_{q2}i_{q2} + v_{d2}i_{d2})e^{2j\phi}$

3. If the abc variables are transformed by two transformations $C(\gamma_1)$ and $C(\gamma_2)$ to two sets of variables, $(d_1, q_1, 0)$ and $(d_2, q_2, 0)$, where $\gamma_1 = \gamma_2 + \phi$, then show that :

$$(f_{q1} + jf_{d1}) = (f_{q2} + jf_{d2})e^{-j\phi}$$

4. Consider a balanced set of star connected 3 phase elements. The transformation of Eqn. 1 is used to transform from abc to dq variables. In steady state $\sqrt{v_d^2 + v_q^2}$ and $\sqrt{i_d^2 + i_q^2}$ are equal to :

   (a) L-L rms voltage and rms current
   (b) L-N rms voltage and $\sqrt{3}\times$ rms current
   (c) L-L rms voltage and $\sqrt{3}\times$ rms current
   (d) L-N rms voltage and rms current

5. If a particular program uses the modeling convention $k_{d1} = 1$, $k_{q1} = -1$ while another uses $k_{d2} = \sqrt{\frac{2}{3}}$, $k_{q2} = \sqrt{\frac{2}{3}}$ (k_d and k_q are explained in Video Lecture-14), then

   (a) $i_{d1} = \sqrt{\frac{2}{3}}i_{d2}$, $i_{q1} = -\sqrt{\frac{2}{3}}i_{q2}$
   (b) $i_{d1} = -\sqrt{\frac{2}{3}}i_{d2}$, $i_{q1} = \sqrt{\frac{2}{3}}i_{q2}$
   (c) $i_{d1} = \sqrt{\frac{3}{2}}i_{d2}$, $i_{q1} = \sqrt{\frac{3}{2}}i_{q2}$
   (d) $i_{d1} = \sqrt{\frac{3}{2}}i_{d2}$, $i_{q1} = -\sqrt{\frac{3}{2}}i_{q2}$

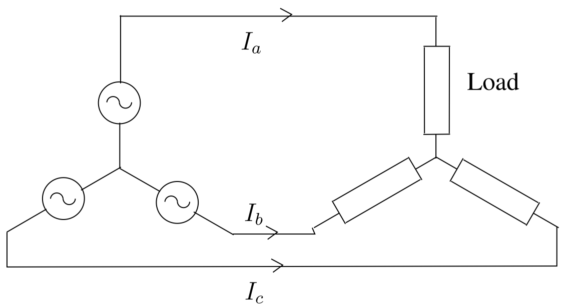
6. Derive the relation-ship (for small deviations) between the polar and rectangular co-ordinates as given below:

$$\begin{bmatrix} \Delta V_Q \\ \Delta V_D \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} \frac{\Delta V}{V} \\ \Delta\theta \end{bmatrix}$$

   where $V = \sqrt{V_Q^2 + V_D^2}$ and $\theta = \tan^{-1}\frac{V_D}{V_Q}$

7. The expression (in terms of d-q variables) of steady state reactive power absorbed by the balanced three phase system shown below is :

   (a) $v_q i_d + v_d i_q$
   (b) $-v_q i_d + v_d i_q$
   (c) $-v_q i_d - v_d i_q$
   (d) $v_q i_d - v_d i_q$



8. Suppose V_a, V_b and V_c are sinusoids with frequency w_o and have a negative sequence component (but no zero sequence component). Then, if the transformation $C(\gamma)$ is used with $\gamma = w_o t + \delta$, where δ is a constant :
(a) V_d and V_q will have sinusoidal components of double the frequency
(b) V_d and V_q will be constants
(c) V_d and V_q will have sinusoidal components of half the frequency
(d) V_d and V_q will be exponentially decaying

9. The effect of neglecting stator transients in the synchronous machine equations during short circuit is equivalent to
(a) neglecting an exponentially decaying component in the stator phase currents
(b) neglecting a high frequency ($50Hz$ decaying oscillatory component in the d & q currents
(c) neglecting an exponentially decaying mode in the field current
(d) neglecting a high frequency ($50Hz$) decaying oscillatory component in the field current
(e) neglecting an exponentially decaying component in electromagnetic torque
(f) neglecting a high frequency decaying oscillatory component in the electromagnetic torque

10. By making appropriate assumptions in the 2.2 model and with $x_d^{''} \approx x_q^{''}$, show that the time constant of decay for the mode referred in the previous question (also called armature time constant (T_a) is ::

$$T_a \approx \frac{x_d^{''}}{\omega_B R_a}$$

11. For obtaining the classical model from a more realistic 2.2 model, the following assumptions have to be made:
(a) The damper windings are opened (i.e, $R_g = R_k = R_h = \infty$) or equivalently $T_q' = T_q'' = T_d'' = 0$
(b) The damper windings have $R_g = R_k = R_h = 0$ or equivalently $T_q' = T_q'' = T_d'' = \infty$
(c) Transient Saliency is neglected: $x_d' = x_q'$
(d) $T_d' = \infty$ or equivalently $\frac{d\psi_f}{dt} = 0$, i.e ψ_f is constant.
(e) $T_d' = 0$ or equivalently $-\psi_f + \psi_d + \frac{x_d'}{x_d - x_d'} E_{fd} = 0$

12. Which of the following set of data on machine MVA base is/are likely to be incorrect ?

Generator	x_d	x_d'	x_d''	T_{do}'	T_{do}''	x_q	x_q'	x_q''	T_{qo}'	T_{qo}''	H
(A)	1.06	0.35	0.25	6.37	0.09	0.66	-	0.3	-	0.09	3.82
(B)	2.31	0.267	0.223	9	0.06	2.19	0.7	0.223	1	0.06	3.0
(C)	2.31	0.267	0.223	9	0.06	2.19	0.7	0.223	1	0.06	0.5
(D)	2.31	0.267	0.223	1	0.06	2.19	0.7	0.223	1	0.06	3.0

13. Consider the system shown in Fig. 1 which consists of 2 *identical* generators within a power plant connected to a large generator via identical transformers and a transmission line. Note that, $P_1,\ P_2,\ P_3$ are the electrical power outputs of generators and $\theta_1,\ \theta_2,\ \theta_3,\ \theta_4$ are the bus voltage phase angles of the respective buses. Answer the following questions : (Assumption:

The identical generators are also operated with the same amount of mechanical power input and field voltage)
(a) How many swing modes are present ?
(b) In which of the variables, $\theta_1, \theta_2, \theta_3, \theta_4$ is the *intra*-plant mode *not observable*
(c) In which of the variables $P_1, P_2, P_3, (P_1 - P_2)$ is the (*inter*-plant) mode not observable.
(d) In which of the variables $P_1, P_2, (\theta_1 - \theta_2)$ is the center of inertia (COI) mode poorly observable.

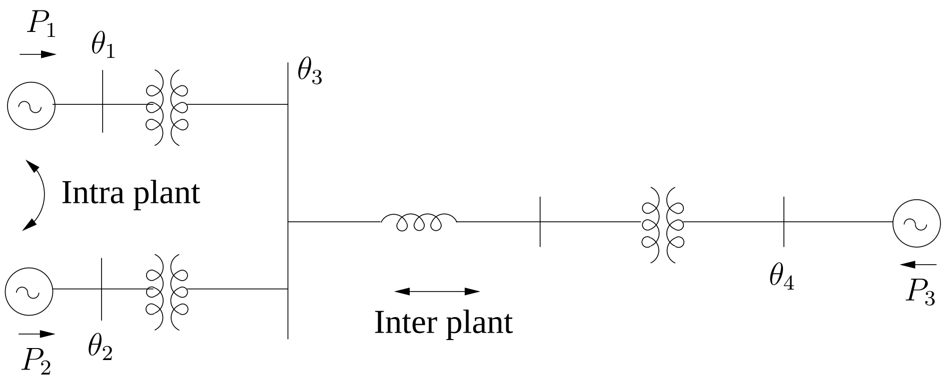


Figure 1:

14. The eigenvalues of a multimachine system are shown below. The generator model 1.1 (two rotor windings are modelled: field winding and one damper winding on the q-axis) is used and excitation system is modelled by a single transfer function. There are no other *dynamical equations*. Answer the following questions :
a) The number of machines which have been considered are : ......
b) The number of swing modes are : ......
c) Are loads frequency dependent/ mechanical damping is present (yes or no)? : ......
Give reason for your answer

Full System Matrix
$-\mathbf{0.8492 \pm j12.7672}$
$-\mathbf{0.2512 \pm j8.3648}$
$-2.2421 \pm j3.0195$
-0.1384
$-4.6654 \pm j1.3830$
$-3.4855 \pm j1.0014$
-2.2614
-0.8882
-3.2258
0.0

15. Consider a *salient pole* synchronous machine whose field winding and damper windings are removed. Also, eddy currents in the rotor are assumed to be negligible and the generator is operating at a constant speed (ω_B). Therefore the only differential equations of the machine

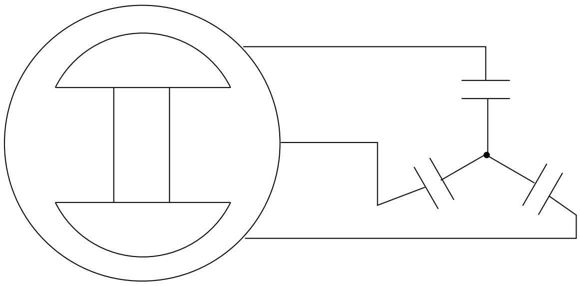


Figure 2:

are those corresponding to the stator d,q fluxes. The generator is connected to a balanced three phase star connected capacitor bank as shown in Fig. 2 .

(a) Write down the differential equations of the machine stator flux and the capacitor voltages in the d-q frame of reference - you need not write the zero sequence differential equations. Do not neglect the stator resistance.

(b) Write the A matrix (state space form).

(c) If x_d and x_q are constant, how may equilibrium points are there ? What is/are the equilibrium value of the states ?

(d) Compute the eigenvalues for the following cases:

  i. $x_d = 1.0$ pu, $x_q = 0.6$ pu, $R_a = 0.01$ pu, $b_c = 0.5$ pu, $\omega_B = 2\pi 50$ rad/s

  ii. $x_d = 1.0$ pu, $x_q = 0.6$ pu, $R_a = 0.01$ pu, $b_c = 1.5$ pu, $\omega_B = 2\pi 50$ rad/s

  iii. Suppose $x_d = x_q$ and we define:
  $W = \frac{x_d}{\omega_B}(i_d^2 + i_q^2) + \frac{b_c}{\omega_B}(v_d^2 + v_q^2)$, then evaluate the expression for $\dfrac{dW}{dt}$. What can you infer from this ?

(e) Comment on the results that you have obtained

16. Starting from a synchronous generator model, derive the model of induction machine. Show that an induction machine driven by prime mover can self excite if a capacitor bank of large enough rating is connected to its terminals.

17. For a balanced three phase electrical network, the relationship between the phase-neutral voltages and currents are given by :

$$\begin{bmatrix} v_a(s) \\ v_b(s) \\ v_c(s) \end{bmatrix} = \begin{bmatrix} Z(s) & 0 & 0 \\ 0 & Z(s) & 0 \\ 0 & 0 & Z(s) \end{bmatrix} \begin{bmatrix} i_a(s) \\ i_b(s) \\ i_c(s) \end{bmatrix}$$

The relationship in the d-q variables is given by :

$$\begin{bmatrix} v_d(s) \\ v_q(s) \end{bmatrix} = \begin{bmatrix} Z_{dd}(s) & Z_{dq}(s) \\ Z_{qd}(s) & Z_{qq}(s) \end{bmatrix} \begin{bmatrix} i_d(s) \\ i_q(s) \end{bmatrix}$$

Derive the expressions for $Z_{dd}(s)$, $Z_{dq}(s)$, $Z_{qd}(s)$ and $Z_{qq}(s)$ in terms of $Z(s)$.
The transformation described by equation 1 is used with $\gamma = \omega_0 t$, where ω_0 is a constant.

18. If a generator is modeled by 1.1 model (two rotor windings are modelled: field winding and one damper winding on the q-axis), the equation for the rotor windings are given by:

$$\frac{d\psi_f}{dt} + R_f i_f = v_f \quad \text{(field winding)}$$

$$\frac{d\psi_g}{dt} + R_g i_g = 0 \qquad \text{(q axis damper)}$$

and the inductance matrix is given by

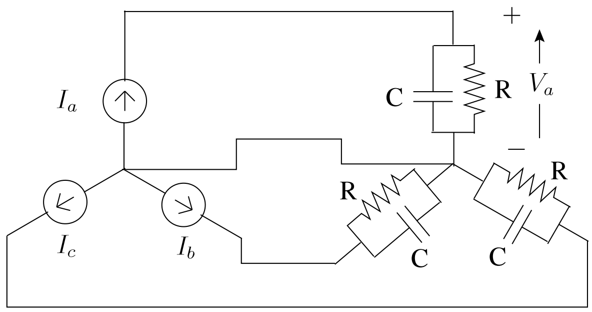
$$\begin{bmatrix} \psi_d \\ \psi_f \\ \psi_q \\ \psi_g \end{bmatrix} = \begin{bmatrix} L_d & M_{df} & 0 & 0 \\ M_{df} & L_{ff} & 0 & 0 \\ 0 & 0 & L_q & M_{qg} \\ 0 & 0 & M_{qg} & L_{gg} \end{bmatrix} \begin{bmatrix} \psi_d \\ \psi_f \\ \psi_q \\ \psi_g \end{bmatrix}$$

(a) show that
$\psi_d(s)$ can be expressed as follows
$\psi_d(s) = L_d(s)I_d(s) + G'(s)V_f(s)$
Write down the expressions for $L_d(s)$ and $G'(s)$ in terms of L_d, M_{df}, L_{ff} and R_f

(b) Similarly show that
$\psi_q(s) = L_q(s)I_q(s)$, write down the expression for $L_q(s)$

(c) Also show that $i_f(s)$ can be written as
$i_f(s) = A_1(s)i_d(s) + A_2(s)v_f(s)$
write down the expression for $A_1(s)$ and $A_2(s)$

19. Write the dynamical equations of the capacitor voltages in the dq0 frame of reference using the transformation of Equation (1) where $\gamma = \omega t$.



$\frac{dV_a}{dt} = -\frac{V_a}{RC} + \frac{I_a}{C}$
and similarly for the other two phases

If the time constant of the circuit is $\tau = RC$, what are the eigenvalues of the system which you have formulated in the dq0 variables ?

20. For each of the following phenomena, indicate the *simplest models* that can be used to get reasonably accurate answers.

Phenomena/ Study	Machine	Network	Excitation	Prime Mover	Load	Mech. Eqns.	Small Signal (linearized) or Nonlinear Models ?
a) Sub-Synchronous Resonance							
b) HVDC power electronic simulation							
c) Power Oscillation damping control design							
d) Islanding and u/f load shedding							
e) Transient Stability							
f) Voltage Stability							

Machine Model:
(A) Constant voltage source
(B) 0.0 (classical model)
(C) 1.0 - stator transients neglected
(D) Damper windings modelled - stator transients neglected
(E) Damper windings modelled -stator transients considered

Network Model:
(A) Lumped parameter dynamical representation
(B) Distributed parameter dynamic representation
(C) Quasi-sinusoidal steady state representation (dynamics neglected)

Excitation System Model
(A) Exciter not modelled (constant excitation voltage)
(B) Detailed excitation system with voltage regulator– over-excitation limiters not modelled
(C) Detailed excitation system with voltage regulator– over-excitation limiters modelled

Prime Mover Model
(A) Not modelled (constant mechanical power)
(B) Simple Model (linear turbine-governor model)
(C) Detailed Model (turbine/ governor/ boiler controls modelled in detail)

Load Model
(A) Static - frequency / voltage dependent loads
(B) Dynamic - large controlled loads and tap changing transformers modelled

Mechanical Equations of Turbine-Generator-Exciter (TGE)

(A) TGE modelled as a single rigid mass
(B) Masses (lumped) connected by elastic shaft model

21. Do the following simulation :

(a) A generator is operating on open circuit at a speed 314 rad/s $= \omega_0$.
At time $t = 5\,s$ the field voltage is 'switched on', i.e. , E_{fd} is given a unit step. Compute the response of the various variables i.e, i_d, i_q, v_d, v_q, ψ_d, ψ_q. Assume that the generator is rotating at a constant speed. Initial values ($t = 0\ s$) of all states are zero.

(b) At $t = 30\,s$, the generator output is shorted. Compute the response of all the variables.

Note: You may take $v_d = Ri_d$ and $v_q = Ri_q$. For open circuit conditions, take $R = 100$ pu and for short circuit $R = 0.01$ pu.
$x_d = 1.79$, $x'_d = 0.169$, $x''_d = 0.135$, $T'_{d0} = 4.3\,s$, $T''_{d0} = 0.032\,s$, $T''_{dc} = 0.032\,s$, $x_q = 1.71$, $x'_q = 0.228$, $x''_q = 0.2$, $T'_{q0} = 0.85\,s$, $T''_{q0} = 0.05\,s$

(c) Repeat the same exercise with stator transients neglected.