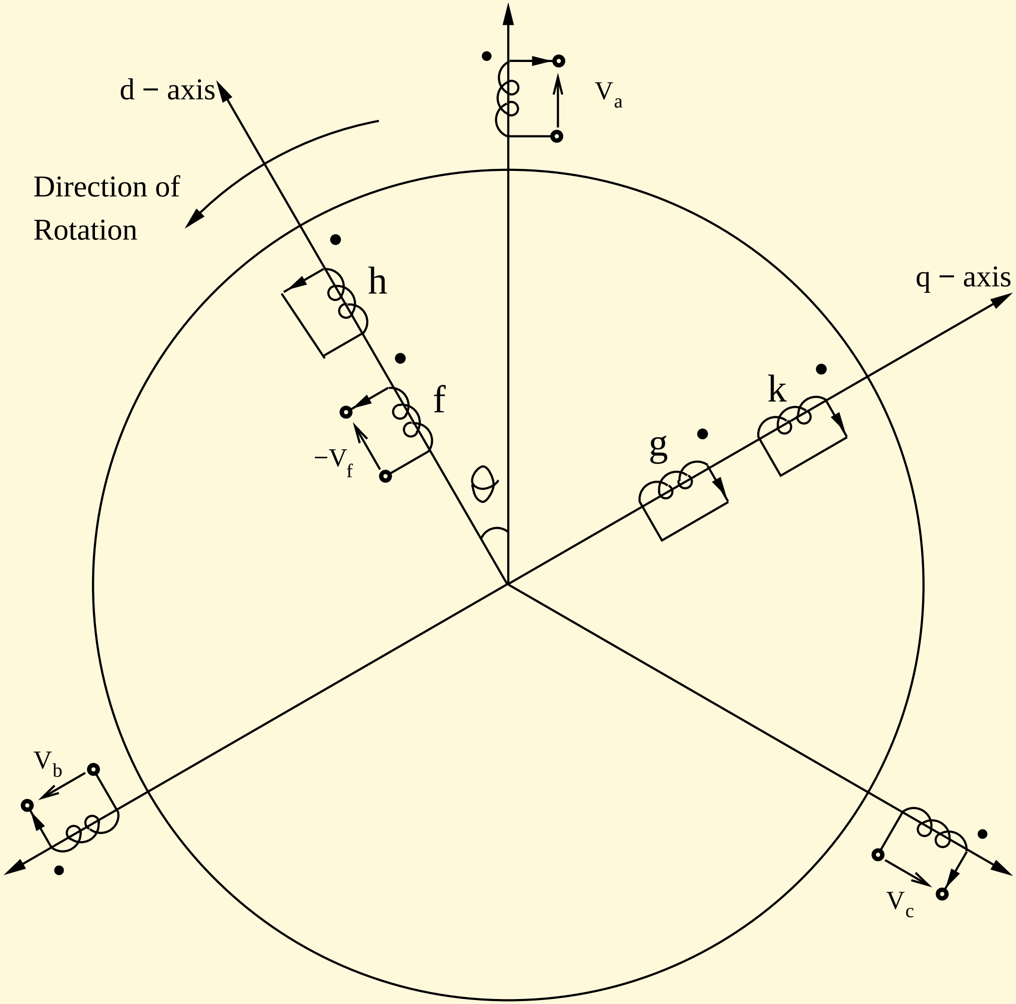
Review of Basic Synchronous Machine Equations



Synchronous machine model - 2 pole

Flux-Current Relationship

The stator and rotor flux linkages are given by

$$\psi_s = [L_{ss}]i_s + [L_{sr}]i_r$$

$$\psi_r = [L_{rs}]i_s + [L_{rr}]i_r$$

where

$$i_s^T = [i_a \ i_b \ i_c], \qquad \psi_s^T = [\psi_a \ \psi_b \ \psi_c]$$

$$i_r^T = [i_f \ i_h \ i_g \ i_k] \qquad \psi_r^T = [\psi_f \ \psi_h \ \psi_g \ \psi_k]$$

$$[L_{ss}] = [L_{ss}]^T$$

$$[L_{rr}] = [L_{rr}]^T$$

$$[L_{rs}] = [L_{sr}]^T$$

$$\left[\, L_{ss} \,\right] = \begin{bmatrix} L_{aao} & L_{abo} & L_{abo} \\ L_{abo} & L_{aao} & L_{abo} \\ L_{abo} & L_{abo} & L_{aao} \end{bmatrix} + L_{aa2} \times$$

$$\begin{bmatrix} \cos 2\theta & \cos(2\theta - 2\pi/3) & \cos(2\theta + 2\pi/3) \\ \cos(2\theta - 2\pi/3) & \cos(2\theta + 2\pi/3) & \cos 2\theta \\ \cos(2\theta + 2\pi/3) & \cos 2\theta & \cos(2\theta - 2\pi/3) \end{bmatrix}$$

θ : Electrical Angle $= \frac{P}{2}\theta_m$

$$\begin{bmatrix} L_{rr} \end{bmatrix} = \begin{bmatrix} L_f & L_{fh} & 0 & 0 \\ L_{fh} & L_h & 0 & 0 \\ 0 & 0 & L_g & L_{gk} \\ 0 & 0 & L_{gk} & L_k \end{bmatrix}$$

$$[\, L_{sr} \,] = [L_{sr}^{d} \quad L_{sr}^{q}]$$

$$[L_{sr}]^{d} = \begin{bmatrix} M_{af}\cos\theta & M_{ah}\cos\theta \\ M_{af}\cos(\theta - 2\pi/3) & M_{ah}\cos(\theta - 2\pi/3) \\ M_{af}\cos(\theta + 2\pi/3) & M_{ah}\cos(\theta + 2\pi/3) \end{bmatrix}$$

$$[L_{sr}]^{q} = \begin{bmatrix} M_{ag}\sin\theta & M_{ak}\sin\theta \\ M_{ag}\sin(\theta - 2\pi/3) & M_{ak}\sin(\theta - 2\pi/3) \\ M_{ag}\sin(\theta + 2\pi/3) & M_{ak}\sin(\theta + 2\pi/3) \end{bmatrix}$$

Voltage Equations

$$-\frac{d\psi_s}{dt} - [R_s]i_s = v_s$$

$$-\frac{d\psi_r}{dt} - [R_r]i_r = v_r$$

where

$$v_s^T = [v_a \; v_b \; v_c], \qquad v_r^T = [-v_f \; 0 \; 0 \; 0]$$

$$\left[R_s \right] = \begin{bmatrix} R_a & 0 & 0 \\ 0 & R_a & 0 \\ 0 & 0 & R_a \end{bmatrix} = R_a [I_{3\times 3}]$$

$$\left[R_r \right] = \begin{bmatrix} R_f & 0 & 0 & 0 \\ 0 & R_h & 0 & 0 \\ 0 & 0 & R_g & 0 \\ 0 & 0 & 0 & R_k \end{bmatrix}$$

Torque Equation

$$J\frac{d^2\theta_m}{dt^2} = T_m - T_e$$

$$T_e = -\frac{\partial W'}{\partial \theta_m} = -\frac{P}{2}\frac{\partial W'}{\partial \theta} = \frac{P}{2}T_e'$$

$$W' = \frac{1}{2}\left[\, i_s^T \;\; i_r^T \,\right]\begin{bmatrix} L_{ss} & L_{sr} \\ L_{rs} & L_{rr} \end{bmatrix}\begin{bmatrix} i_s \\ i_r \end{bmatrix}$$

$$T_e' = -\frac{1}{2}\left[i_s^T [\frac{\partial L_{ss}}{\partial \theta}] i_s + 2 i_s^T [\frac{\partial L_{sr}}{\partial \theta}] i_r \right]$$