Synchronous Machine Equations in the d-q-o frame

Stator Voltage Equations

$$-\frac{d\psi_d}{dt} - \omega\frac{k_q}{k_d}\psi_q - R_a i_d = v_d$$

$$-\frac{d\psi_q}{dt} + \omega\frac{k_d}{k_q}\psi_d - R_a i_q = v_q$$

$$-\frac{d\psi_o}{dt} - R_a i_o = v_o$$

$$\omega = \frac{d\theta}{dt}$$

Rotor Voltage Equations

$$\frac{d\psi_f}{dt} + R_f i_f = v_f$$

$$\frac{d\psi_h}{dt} + R_h i_h = 0$$

$$\frac{d\psi_g}{dt} + R_g i_g = 0$$

$$\frac{d\psi_k}{dt} + R_k i_k = 0$$

Current-Flux Relationship

$$\begin{bmatrix} \psi_{dqo} \\ \psi_r \end{bmatrix} = \begin{bmatrix} L'_{ss} & L'_{sr} \\ L'_{rs} & L_{rr} \end{bmatrix} \begin{bmatrix} i_{dqo} \\ i_r \end{bmatrix}$$

$$[L'_{ss}] = \begin{bmatrix} L_d & 0 & 0 \\ 0 & L_q & 0 \\ 0 & 0 & L_o \end{bmatrix}$$

$$[L'_{sr}] = \begin{bmatrix} (\frac{M_{af}}{k_d}) & (\frac{M_{ah}}{k_d}) & 0 & 0 \\ 0 & 0 & (\frac{M_{ag}}{k_q}) & (\frac{M_{ak}}{k_q}) \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[L'_{rs}] = \begin{bmatrix} (\frac{3}{2}M_{af}k_d) & 0 & 0 \\ (\frac{3}{2}M_{ah}k_d) & 0 & 0 \\ 0 & (\frac{3}{2}M_{ag}k_q) & 0 \\ \\ 0 & (\frac{3}{2}M_{ak}k_q) & 0 \end{bmatrix}$$

$$\begin{bmatrix} L_{rr} \end{bmatrix} = \begin{bmatrix} L_f & L_{fh} & 0 & 0 \\ L_{fh} & L_h & 0 & 0 \\ 0 & 0 & L_g & L_{gk} \\ 0 & 0 & L_{gk} & L_k \end{bmatrix}$$

Expression for Electrical Torque

$$T_e = \frac{P}{2} T_e'$$

$$= \frac{P}{2} \ \frac{3}{2} \ k_d \ k_q \ [i_q \psi_d - i_d \psi_q]$$

Choice of k_d, k_q and k_o

With $k_d = \sqrt{\frac{2}{3}}, k_q = \sqrt{\frac{2}{3}}$ and $k_o = \sqrt{\frac{1}{3}}$,

1. $[L'_{rs}]^T = [L'_{sr}]$

2. $[C_P]^T = [C_P]^{-1}$ which also implies power invariance (why ?)

3. $T_e = \frac{P}{2}[i_q\psi_d - i_d\psi_q]$