Synchronous Machine Models

Flux and Current Equations on the d axis (From Model - with $k_d = \sqrt{\frac{2}{3}}$)

$$\psi_d = L_d i_d + M_{df} i_f + M_{dh} i_h$$

$$\psi_f = M_{df} i_d + L_{ff} i_f + L_{fh} i_h$$

$$\psi_h = M_{dh} i_d + L_{fh} i_f + L_{hh} i_h$$

$$\frac{d\psi_f}{dt} + R_f i_f = v_f$$

$$\frac{d\psi_h}{dt} + R_h i_h = 0$$

Laplace Transform of Flux and Current Equations on the d axis

$$\Psi_d(s) = L_d I_d(s) + M_{df} I_f(s) + M_{dh} I_h(s)$$

$$\Psi_f(s) = M_{df} I_d(s) + L_{ff} I_f(s) + L_{fh} I_h(s)$$

$$\Psi_h(s) = M_{dh} I_d(s) + L_{fh} I_f(s) + L_{hh} I_h(s)$$

$$s\Psi_f(s) + R_f I_f(s) = V_f(s)$$

$$s\Psi_h(s) + R_h I_h(s) = 0$$

d - axis Flux-Current Transfer Function

Eliminating $\Psi_f(s), \Psi_h(s), I_f(s), I_h(s)$**,**

$$\Psi_d(s) = L_d(s) I_d(s) + G'(s) V_f(s)$$

$$L_d(s) = L_d \frac{(1 + B_N s + A_N s^2)}{(1 + B_D s + A_D s^2)}$$

$$G'(s) = \frac{M_{df}}{R_f} \frac{(1 + A_G s)}{(1 + A_D s + B_D s^2)}$$

$$B_N = \frac{L_{ff}}{R_f} + \frac{L_{hh}}{R_h} - \frac{M_{dh}^2}{L_d R_h} - \frac{M_{df}^2}{L_d R_f}$$

$$A_N = \frac{L_{ff} L_{hh}}{R_f R_h} - \frac{L_{fh}^2}{R_f R_h} - \frac{M_{df}^2 L_{hh}}{L_d R_f R_h}$$

$$- \frac{M_{dh}^2 L_{ff}}{L_d R_f R_h} + 2 \frac{M_{dh} M_{df} L_{fh}}{L_d R_f R_h}$$

$$B_D = \frac{L_{ff}}{R_f} + \frac{L_{hh}}{R_h}, \quad A_D = \frac{L_{ff} L_{hh}}{R_f R_h} - \frac{L_{fh}^2}{R_f R_h}$$

$$A_G = \frac{L_{hh}}{R_h} - \frac{M_{dh} L_{fh}}{M_{df} R_h}$$

Equating with Transfer Function Obtained from Measurement, with $v_f = 0$

$$\frac{\Psi_d(s)}{I_d(s)} = L_d \frac{(1 + sT'_d)(1 + sT''_d)}{(1 + sT'_{do})(1 + sT''_{do})}$$

Gives us L_d and the following relationships:

$$T'_d + T''_d = B_N$$

$$T'_d T''_d = A_N$$

$$T'_{do} + T''_{do} = B_D$$

$$T'_{do} T''_{do} = A_D$$

Equating with Transfer Function Obtained from Measurement, with $I_d = 0$

$$\frac{\Psi_d(s)}{V_f(s)} = \frac{M_{df}}{R_f}\frac{(1 + sT''_{dc})}{(1 + sT'_{do})(1 + sT''_{do})}$$

Gives us $\frac{M_{df}}{R_f}$ and the following relationship:

$$T''_{dc} = A_G$$

Model Parameters on d-axis(Eight):

$$L_d, M_{df}, M_{dh}, L_{ff}, L_{fh}, L_{hh}, R_f, R_h$$

Parameters from **One** Measurement (Five):

$$L_d, T'_d, T''_d, T'_{do}, T''_{do}$$

Note: One cannot get a unique solution for the model parameters with just one transfer function measurement.
Stator resistance also required; can be obtained from a separate measurement.

Flux and Current Equations on the q axis (From Model - with $k_q = \sqrt{\frac{2}{3}}$)

$$\psi_q = L_q i_q + M_{qg} i_g + M_{qk} i_k$$

$$\psi_g = M_{qg} i_q + L_{gg} i_g + L_{gk} i_k$$

$$\psi_k = M_{qk} i_q + L_{gk} i_g + L_{kk} i_k$$

$$\frac{d\psi_g}{dt} + R_g i_g = v_g$$

$$\frac{d\psi_k}{dt} + R_k i_k = 0$$

Laplace Transform of Flux and Current Equations on the q axis

$$\Psi_q(s) = L_q I_q(s) + M_{qg} I_g(s) + M_{qk} I_k(s)$$

$$\Psi_g(s) = M_{qg} I_q(s) + L_{gg} I_g(s) + L_{gk} I_k(s)$$

$$\Psi_k(s) = M_{qk} I_q(s) + L_{gk} I_g(s) + L_{kk} I_k(s)$$

$$s\Psi_g(s) + R_g I_g(s) = V_g(s)$$

$$s\Psi_k(s) + R_k I_k(s) = 0$$

q - axis Flux-Current Transfer Function Eliminating $\Psi_g(s), \Psi_k(s), I_g(s), I_k(s)$,

$$\Psi_q(s) = L_q(s) I_q(s)$$

$$L_q(s) = L_q \frac{(1 + B_{Nq} s + A_{Nq} s^2)}{(1 + B_{Dq} s + A_{Dq} s^2)}$$

$$B_{Nq} = \frac{L_{gg}}{R_g} + \frac{L_{kk}}{R_k} - \frac{M_{qk}^2}{L_q R_k} - \frac{M_{qg}^2}{L_q R_g}$$

$$A_{Nq} = \frac{L_{gg} L_{kk}}{R_g R_k} - \frac{L_{gk}^2}{R_g R_k} - \frac{M_{qg}^2 L_{kk}}{L_q R_g R_k} - \frac{M_{qk}^2 L_{gg}}{L_q R_g R_k} + 2\frac{M_{qk} M_{qg} L_{gk}}{L_q R_g R_k}$$

$$B_{Dq} = \frac{L_{gg}}{R_g} + \frac{L_{kk}}{R_k}$$

$$A_{Dq} = \frac{L_{gg} L_{kk}}{R_g R_k} - \frac{L_{gk}^2}{R_g R_k}$$

Equating with Transfer Function Obtained from Measurement

$$\frac{\Psi_q(s)}{I_q(s)} = L_q \frac{(1+sT_q')(1+sT_q'')}{(1+sT_{qo}')(1+sT_{qo}'')}$$

Gives us L_q and the following relationships:

$$T_q' + T_q'' = B_{Nq}$$

$$T_q' T_q'' = A_{Nq}$$

$$T_{qo}' + T_{qo}'' = B_{Dq}$$

$$T_{qo}' T_{qo}'' = A_{Dq}$$

Model Parameters on q-axis(Eight):

$$L_q, M_{qg}, M_{qk}, L_{gg}, L_{gk}, L_{kk}, R_g, R_k$$

Parameters from only **One** Measurement made on the q-axis (Five):

$$L_q, T_q', T_q'', T_{qo}', T_{qo}''$$

Possibilities

1. Use a state space model which requires fewer parameters - BUT states cannot be easily related to the original states (stator and rotor fluxes). **Model A**

2. Use a state space which requires fewer parameters, making some assumptions, so that the state space model is in terms of states that can be related to the original states easily. The parameters of this model need to be back-calculated from the standard parameters.(**Model I**)

3. Use a state space which requires fewer parameters, making some assumptions, so that the state space model is in terms of states that can be related to the original states easily. Additionally the parameters in the state space model directly use the "standard parameters" without need for back-calculations. (**Model II**)

Model A

A state space model which requires fewer parameters - BUT states cannot be easily related to the original states (stator and rotor fluxes).

q axis Model - Transfer Function

$$\frac{\Psi_q(s)}{I_q(s)} = Lq\frac{(1 + sT'_q)(1 + sT''_q)}{(1 + sT'_{qo})(1 + sT''_{qo})}$$

q axis Model - Alternative Expression of Transfer Function

$$\frac{I_q(s)}{\Psi_q(s)} = \frac{1}{L_q} + \left(\frac{1}{L'_q} - \frac{1}{L_q}\right)\frac{sT'_q}{(1 + sT'_q)} + \left(\frac{1}{L''_q} - \frac{1}{L'_q}\right)\frac{sT''_q}{(1 + sT''_q)}$$

q axis - Standard Parameters from Measurement

$$L_q, T_q', T_q'', T_{qo}', T_{qo}''$$

OR

$$L_q, L_q', L_q'', T_{qo}', T_{qo}''$$

OR

$$L_q, T_q', T_q'', L_q', L_q''$$

NOTE: Stator Resistance can also be obtained by measurement

q axis Model - Inter-relationships between Standard Parameters

$$T'_{qo} + T''_{qo} = \frac{L_q}{L'_q} T'_q + \left(1 - \frac{L_q}{L'_q} + \frac{L_q}{L''_q}\right) T''_q$$

$$T'_{qo} T''_{qo} = T'_q T''_q \frac{L_q}{L''_q}$$

q-axis Model A

$$\frac{d\psi_G}{dt} = \frac{1}{T'_q}(-\psi_G + \psi_q)$$

$$\frac{d\psi_K}{dt} = \frac{1}{T''_q}(-\psi_K + \psi_q)$$

$$\psi_q = L''_q i_q + \frac{(L'_q - L''_q)}{L'_q}\psi_K + \frac{(L_q - L'_q)}{L_q}\frac{L''_q}{L'_q}\psi_G$$

$$\frac{d\psi_q}{dt} = \omega\psi_d - R_a i_q - v_q$$

ψ_G and ψ_K are linearly related to ψ_g and ψ_k.

d axis

d axis Model - Transfer Function

$$\Psi_d(s) = L_d \frac{(1 + sT'_d)(1 + sT''_d)}{(1 + sT'_{do})(1 + sT''_{do})} I_d(s)$$

$$+ \frac{(1 + sT''_{dc})}{(1 + sT'_{do})(1 + sT''_{do})} \frac{M_{df}}{R_f} V_f(s)$$

d axis Model - Alternative Expression of Transfer Function $v_f = 0$

$$\frac{I_d(s)}{\Psi_d(s)} = \frac{1}{L_d} + \left(\frac{1}{L'_d} - \frac{1}{L_d}\right)\frac{sT'_d}{(1 + sT'_d)} + \left(\frac{1}{L''_d} - \frac{1}{L'_d}\right)\frac{sT''_d}{(1 + sT''_d)}$$

d axis - Standard Parameters from Measurement

$$L_d, T'_d, T''_d, T'_{do}, T''_{do}$$

OR

$$L_d, L'_d, L''_d, T'_{do}, T''_{do}$$

OR

$$L_d, T'_d, T''_d, L'_d, L''_d$$

NOTE: Stator Resistance can also be obtained by measurement

d axis Model - Inter-relationships between Standard Parameters

$$T'_{do} + T''_{do} = \frac{L_d}{L'_d} T'_d + \left(1 - \frac{L_d}{L'_d} + \frac{L_d}{L''_d}\right) T''_d$$

$$T'_{do} T''_{do} = T'_d T''_d \frac{L_d}{L''_d}$$

d-axis Model A

$$\frac{d\psi_H}{dt} = \frac{1}{T''_d}(-\psi_H + \psi_d) + \frac{\beta_1}{T''_d} v_f$$

$$\frac{d\psi_F}{dt} = \frac{1}{T'_d}(-\psi_F + \psi_d) + \frac{\beta_2}{T'_d} v_f$$

$$\psi_d = L''_d i_d + \frac{(L'_d - L''_d)}{L'_d}\psi_H + \frac{(L_d - L'_d)}{L_d}\frac{L''_d}{L'_d}\psi_F$$

$$\frac{d\psi_d}{dt} = -\omega\psi_q - R_a i_d - v_d$$

Expressions for β_1 and β_2

$$\beta_1 = \frac{(T''_d - T''_{dc})}{(T''_d - T'_d)} \frac{L'_d L''_d}{L_d(L'_d - L''_d)} \frac{M_{df}}{R_f}$$

$$\beta_2 = \frac{(T'_d - T''_{dc})}{(T'_d - T''_d)} \frac{L'_d}{(L_d - L'_d)} \frac{M_{df}}{R_f}$$

Model I

A state space model using some assumptions, so that the state space model is in terms of states that can be related to the original states easily. The parameters of this model need to be back-calculated from the standard parameters.

Original Equations in the d-axis

$$\psi_d = L_d i_d + M_{df} i_f + M_{dh} i_h$$

$$\psi_f = M_{df} i_d + L_{ff} i_f + L_{fh} i_h$$

$$\psi_h = M_{dh} i_d + L_{fh} i_f + L_{hh} i_h$$

$$\frac{d\psi_f}{dt} + R_f i_f = v_f$$

$$\frac{d\psi_h}{dt} + R_h i_h = 0$$

$$\frac{d\psi_d}{dt} = -\omega\psi_q - R_a i_d - v_d$$

Alternative d-axis variables

Define New Variables (similar to referring variables on to one side of a transformer)

$$\psi'_f = \psi_f \alpha_f$$

$$\psi'_h = \psi_h \alpha_h$$

$$i'_f = \frac{i_f}{\alpha_f}$$

$$i'_h = \frac{i_h}{\alpha_h}$$

Equations in New Variables

$$\psi_d = L_d i_d + M'_{df} i'_f + M'_{dh} i'_h$$

$$\psi'_f = M'_{df} i_d + L'_{ff} i'_f + L'_{fh} i'_h$$

$$\psi'_h = M'_{dh} i_d + L'_{fh} i'_f + L'_{hh} i'_h$$

$$\frac{d\psi'_f}{dt} + R'_f i'_f = v'_f$$

$$\frac{d\psi'_h}{dt} + R'_h i'_h = 0$$

$$\frac{d\psi_d}{dt} = -\omega\psi_q - R_a i_d - v_d$$

Equations in New Variables

$$M'_{df} = \alpha_f M_{df}, \quad M'_{dh} = \alpha_h M_{dh}$$

$$L'_{ff} = \alpha_f^2 L_{ff}, \quad L'_{fh} = \alpha_h \alpha_f L_{fh}$$

$$R'_f = \alpha_f^2 R_f, \quad v'_f = \alpha_f v_f$$

$$R'_h = \alpha_h^2 R_h \quad L'_{hh} = \alpha_h^2 L_{hh}$$

Simplifications to Reduce Number of Parameters

1. Choose α_h so that $M'_{dh} = M'_{df}$
2. If α_f is the actual turns ratio between stator and field winding, then $M'_{df} = L_d - L_l$, L_l is a leakage inductance.
3. **Assume** $M'_{df} = L'_{fh}$

Back-calculation of Required Parameters

NOTE: Form of the transfer functions is unchanged:

$$\Psi_d(s) = L_d(s) I_d(s) + G'(s) V_f'(s)$$

$$L_d(s) = L_d \frac{(1 + B_N s + A_N s^2)}{(1 + B_D s + A_D s^2)}$$

$$G'(s) = \frac{M'_{df}}{R'_f} \frac{(1 + A_G s)}{(1 + A_D s + B_D s^2)}$$

$$B_N = T'_d + T''_d = \frac{L'_{ff}}{R'_f} + \frac{L'_{hh}}{R'_h} - \frac{{M'_{dh}}^2}{L_d R'_h} - \frac{{M'_{df}}^2}{L_d R'_f}$$

$$A_N = T'_d T''_d = \frac{L'_{ff} L'_{hh}}{R'_f R'_h} - \frac{{L'_{fh}}^2}{R'_f R'_h} - \frac{{M'_{df}}^2 L'_{hh}}{L_d R'_f R'_h}$$

$$- \frac{{M'_{dh}}^2 L'_{ff}}{L_d R'_f R'_h} + 2 \frac{M'_{dh} M'_{df} L'_{fh}}{L_d R'_f R'_h}$$

$$B_D = T'_{do} + T''_{do} = \frac{L'_{ff}}{R'_f} + \frac{L'_{hh}}{R'_h}$$

$$A_D = T'_{do} T''_{do} = \frac{L'_{ff} L'_{hh}}{R'_f R'_h} - \frac{{L'_{fh}}^2}{R'_f R'_h}$$

Thus, one may obtain $L'_{ff}, L'_{hh}, R'_f, R'_h$, given

$$L_d, T'_d, T''_d, T'_{do}, T''_{do}, L_l$$

Parameters for this Model (with assumptions):

$$L_d, L'_{ff}, L'_{hh}, R'_f, R'_h, L_l$$

Parameters from measurement:

$$L_d, T'_d, T''_d, T'_{do}, T''_{do}, L_l$$

R_a is available from measurement.

α_f is not explicitly required if referred voltage v'_f is used in all calculations.

Summary: Model I (d axis)

$$\psi_d = L_d i_d + (L_d - L_l) i'_f + (L_d - L_l) i'_h$$

$$\psi'_f = (L_d - L_l) i_d + L'_{ff} i'_f + (L_d - L_l) i'_h$$

$$\psi'_h = (L_d - L_l) i_d + (L_d - L_l) i'_f + L'_{hh} i'_h$$

$$\frac{d\psi'_f}{dt} + R'_f i'_f = v'_f$$

$$\frac{d\psi'_h}{dt} + R'_h i'_h = 0$$

$$\frac{d\psi_d}{dt} = -\omega\psi_q - R_a i_d - v_d$$

Parameters for this Model (with assumptions):

$$L_d, L'_{ff}, L'_{hh}, R'_f, R'_h, L_l$$

Parameters from measurement:

$$L_d, T'_d, T''_d, T'_{do}, T''_{do}, L_l$$

R_a is available from measurement.

α_f is not explicitly required if referred voltage v'_f is used in all calculations.

Summary: Model I (q axis)

$$\psi_q = L_q i_q + (L_q - L_l) i'_g + (L_q - L_l) i'_k$$

$$\psi'_g = (L_q - L_l) i_q + L'_{gg} i'_g + (L_q - L_l) i'_k$$

$$\psi'_k = (L_q - L_l) i_q + (L_q - L_l) i'_g + L'_{kk} i'_k$$

$$\frac{d\psi'_g}{dt} + R'_g i'_g = 0$$

$$\frac{d\psi'_k}{dt} + R'_k i'_k = 0$$

$$\frac{d\psi_q}{dt} = \omega \psi_d - R_a i_q - v_q$$

Parameters for this Model :

$$L_q, L'_{gg}, L'_{kk}, R'_g, R'_k, L_l$$

Parameters from measurement:

$$L_q, T'_q, T''_q, T'_{qo}, T''_{qo}, L_l$$

R_a is available from measurement.

Model I Equivalent Circuits

Model II

A state space model using some assumptions, so that the state space model is in terms of states that can be related to the original states approximately. The parameters of this model do not need to be back-calculated from the standard parameters.

d-axis variables

Define New Variables (similar to referring variables on to one side of a transformer)

$$\psi_F = \psi_F \alpha_F$$

$$\psi_H = \psi_H \alpha_H$$

$$i_F = \frac{i_F}{\alpha_F}$$

$$i_H = \frac{i_H}{\alpha_H}$$

Equations in New Variables

$$\psi_d = L_d i_d + M'_{df} i_F + M'_{dh} i_H$$

$$\psi_F = M'_{df} i_d + L'_{ff} i_F + L'_{fh} i_H$$

$$\psi_H = M'_{dh} i_d + L'_{fh} i_F + L'_{hh} i_H$$

$$\frac{d\psi_F}{dt} + R'_f i_F = v_F$$

$$\frac{d\psi_H}{dt} + R'_h i_H = 0$$

$$\frac{d\psi_d}{dt} = -\omega\psi_q - R_a i_d - v_d$$

Equations in New Variables

$$M'_{df} = \alpha_F M_{df}, \quad M'_{dh} = \alpha_H M_{dh}$$

$$L'_{ff} = \alpha_F^2 L_{ff}, \quad L'_{fh} = \alpha_H \alpha_F L_{fh}$$

$$R'_f = \alpha_F^2 R_f, \quad v'_f = \alpha_F v_f$$

$$R'_h = \alpha_H^2 R_h \quad L'_{hh} = \alpha_H^2 L_{hh}$$

Assumptions to Reduce Number of Parameters

1. Choose α_H and α_F so that $M'_{dh} = M'_{df} = L_d$
2. α_F is the not the actual turns ratio between stator and field winding (but approximately equal to it).
3. **Assume** $L_d = L'_{fh}$

Interesting Observations

The assumption made in this model leads us to:

1. $T''_{dc} = T''_{d}$
2. $T''_{d} = \frac{L'_{hh} - L_d}{R'_h}$
3. $T'_{d} = \frac{L'_{ff} - L_d}{R'_f}$
4. $L'_{d} = \frac{L_d}{L'_{ff}}(L'_{ff} - L_d)$
5. $L''_{d} = \frac{L_d}{L'_{hh}}(L'_{hh} - L_d)$

Equations in New Variables

$$\psi_d = L_d i_d + L_d i_F + L_d i_H$$

$$\psi_F = L_d i_d + L'_{ff} i_F + L_d i_H$$

$$\psi_H = L_d i_d + L_d i_F + L'_{hh} i_H$$

$$\frac{d\psi_F}{dt} + R'_f i_F = v'_f$$

$$\frac{d\psi_H}{dt} + R'_h i_H = 0$$

$$\frac{d\psi_d}{dt} = -\omega\psi_q - R_a i_d - v_d$$

Parameters for this Model :

$$L_d, L'_{ff}, L'_{hh}, R'_f, R'_h$$

Parameters from measurement:

$$L_d, T'_d, T''_d, T'_{do}, T''_{do}$$

R_a is available from measurement.

d-axis Model II

$$\frac{d\psi_H}{dt} = \frac{1}{T''_d}(-\psi_H + \psi_d)$$

$$\frac{d\psi_F}{dt} = \frac{1}{T'_d}(-\psi_F + \psi_d) + \frac{L'_d M'_{df}}{(L_d - L'_d)R'_f T'_d} v'_f$$

$$\psi_d = L''_d i_d + \frac{(L'_d - L''_d)}{L'_d}\psi_H + \frac{(L_d - L'_d)}{L_d}\frac{L''_d}{L'_d}\psi_F$$

$$\frac{d\psi_d}{dt} = -\omega\psi_q - R_a i_d - v_d$$

$$\psi_F = L_d \; i_d + (L_d + \frac{L_d \; L'_d}{L_d - L'_d}) \; i_F + L_d \; i_H$$

$$\psi_H = L_d \; i_d + L_d \; i_F + (L_d + \frac{L'_d \; L''_d}{L'_d - L''_d}) \; i_H$$

q-axis Model II

$$\frac{d\psi_G}{dt} = \frac{1}{T_q'}(-\psi_G + \psi_q)$$

$$\frac{d\psi_K}{dt} = \frac{1}{T_q''}(-\psi_K + \psi_q)$$

$$\psi_q = L_q'' i_q + \frac{(L_q' - L_q'')}{L_q'}\psi_K + \frac{(L_q - L_q')}{L_q}\frac{L_q''}{L_q'}\psi_G$$

$$\frac{d\psi_q}{dt} = \omega\psi_d - R_a i_q - v_q$$

Model II Equivalent Circuits

Per-Unit System

Definition of Base Values

1. V_B = Rated Line-Line Voltage,
2. MVA_B = Rated MVA (three phase)
3. ω_B= Rated (electrical) frequency (rad/s)
4. $I_B = \frac{MVA_B}{V_B}, \omega_{mB} = \frac{2}{P}\omega_B$
5. $T_B = \frac{MVA_B}{\omega_{mB}}, \psi_B = \frac{V_B}{\omega_B}$
6. $Z_B = \frac{V_B}{I_B}, L_B = \frac{Z_B}{\omega_B}$

Model I -PER UNIT

$$\psi_d = x_d i_d + (x_d - x_l) i'_f + (x_d - x_l) i'_h$$

$$\psi'_f = (x_d - x_l) i_d + x'_{ff} i'_f + (x_d - x_l) i'_h$$

$$\psi'_h = (x_d - x_l) i_d + (x_d - x_l) i'_f + x'_{hh} i'_h$$

$$\frac{d\psi'_f}{dt} + \omega_B R'_f i'_f = \frac{\omega_B v'_f}{V_B}$$

$$\frac{d\psi'_h}{dt} + \omega_B R'_h i'_h = 0$$

$$\frac{d\psi_d}{dt} = -\omega \psi_q - \omega_B R_a i_d - \omega_B v_d$$

ω and ω_B are in rad/s, x in per unit

Model II - PER UNIT

per-unit stator line to line voltage under open circuit conditions

$$E_{fd} = \frac{x'_{df}}{R'_f} \frac{v'_f}{V_B}$$

$$x'_{df} = x_d - x_l$$

Zero Sequence Equations in pu

(not required for balanced situations)

$$\frac{d\psi_o}{dt} = \omega_B R_a i_o - \omega_B v_o$$

Model I -PER UNIT

$$\psi_q = x_q i_q + (x_q - x_l) i'_g + (x_q - x_l) i'_k$$

$$\psi'_g = (x_q - x_l) i_q + x'_{gg} i'_g + (x_q - x_l) i'_k$$

$$\psi'_k = (x_q - x_l) i_d + (x_q - x_l) i'_g + x'_{kk} i'_k$$

$$\frac{d\psi'_g}{dt} + R'_g i'_g = 0$$

$$\frac{d\psi'_k}{dt} + R'_k i'_k = 0$$

$$\frac{d\psi_q}{dt} = \omega\psi_d - \omega_B R_a i_q - \omega_B v_q$$

ω and ω_B are in rad/s, x in per unit

Model II - PER UNIT

$$\frac{d\psi_H}{dt} = \frac{1}{T''_d}(-\psi_H + \psi_d)$$

$$\frac{d\psi_F}{dt} = \frac{1}{T'_d}(-\psi_F + \psi_d + \frac{x'_d}{(x_d - x'_d)}E_{fd})$$

$$\psi_d = x''_d i_d + \frac{(x'_d - x''_d)}{x'_d}\psi_H + \frac{(x_d - x'_d)}{x_d}\frac{x''_d}{x'_d}\psi_F$$

$$\frac{d\psi_d}{dt} = -\omega\psi_q - \omega_B R_a i_d - \omega_B v_d$$

ω and ω_B are in rad/s

Model II - PER UNIT

E_{fd} = line-line open circuit voltage in per-unit

Zero Sequence Equations in pu

(not required for balanced situations)

$$\frac{d\psi_o}{dt} = \omega_B R_a i_o - \omega_B v_o$$

Model II - PER UNIT

$$\frac{d\psi_G}{dt} = \frac{1}{T_q'}(-\psi_G + \psi_q)$$

$$\frac{d\psi_K}{dt} = \frac{1}{T_q''}(-\psi_K + \psi_q)$$

$$\psi_q = x_q'' i_q + \frac{(x_q' - x_q'')}{x_q'}\psi_K + \frac{(x_q - x_q')}{x_q}\frac{x_q''}{x_q'}\psi_G$$

$$\frac{d\psi_q}{dt} = \omega\psi_d - \omega_B R_a i_q - \omega_B v_q$$

ω and ω_B are in rad/s

Torque Equation in per-unit

$$\frac{2H}{\omega_B}\frac{d\omega}{dt} = T_m - (\psi_d i_q - \psi_q i_d)$$

ω is the electrical angular speed in rad/s.

ω_B is the base electrical angular speed in rad/s.