Synchronous Machine Models

Model A

A state space model which requires fewer parameters - BUT states cannot be easily related to the original states (stator and rotor fluxes).

q axis - Standard Parameters from Measurement

$$L_q, T'_q, T''_q, T'_{qo}, T''_{qo}$$

OR

$$L_q, L'_q, L''_q, T'_{qo}, T''_{qo}$$

OR

$$L_q, T'_q, T''_q, L'_q, L''_q$$

NOTE: Stator Resistance can also be obtained by measurement

q axis Model - Inter-relationships between Standard Parameters

$$T'_{qo} + T''_{qo} = \frac{L_q}{L'_q} T'_q + \left(1 - \frac{L_q}{L'_q} + \frac{L_q}{L''_q}\right) T''_q$$

$$T'_{qo} T''_{qo} = T'_q T''_q \frac{L_q}{L''_q}$$

q-axis Model A

$$\frac{d\psi_G}{dt} = \frac{1}{T'_q}(-\psi_G + \psi_q)$$

$$\frac{d\psi_K}{dt} = \frac{1}{T''_q}(-\psi_K + \psi_q)$$

$$\psi_q = L''_q i_q + \frac{(L'_q - L''_q)}{L'_q}\psi_K + \frac{(L_q - L'_q)}{L_q}\frac{L''_q}{L'_q}\psi_G$$

$$\frac{d\psi_q}{dt} = \omega\psi_d - R_a i_q - v_q$$

ψ_G and ψ_K are linearly related to ψ_g and ψ_k.

d axis - Standard Parameters from Measurement

$$L_d, T'_d, T''_d, T'_{do}, T''_{do}$$

OR

$$L_d, L'_d, L''_d, T'_{do}, T''_{do}$$

OR

$$L_d, T'_d, T''_d, L'_d, L''_d$$

NOTE: Stator Resistance can also be obtained by measurement

d axis Model - Inter-relationships between Standard Parameters

$$T'_{do} + T''_{do} = \frac{L_d}{L'_d} T'_d + \left(1 - \frac{L_d}{L'_d} + \frac{L_d}{L''_d}\right) T''_d$$

$$T'_{do} T''_{do} = T'_d T''_d \frac{L_d}{L''_d}$$

d-axis Model A

$$\frac{d\psi_H}{dt} = \frac{1}{T''_d}(-\psi_H + \psi_d) + \frac{\beta_1}{T''_d} v_f$$

$$\frac{d\psi_F}{dt} = \frac{1}{T'_d}(-\psi_F + \psi_d) + \frac{\beta_2}{T'_d} v_f$$

$$\psi_d = L''_d i_d + \frac{(L'_d - L''_d)}{L'_d}\psi_H + \frac{(L_d - L'_d)}{L_d}\frac{L''_d}{L'_d}\psi_F$$

$$\frac{d\psi_d}{dt} = -\omega\psi_q - R_a i_d - v_d$$

Expressions for β_1 and β_2

$$\beta_1 = \frac{(T''_d - T''_{dc})}{(T''_d - T'_d)} \frac{L'_d L''_d}{L_d (L'_d - L''_d)} \frac{M_{df}}{R_f}$$

$$\beta_2 = \frac{(T'_d - T''_{dc})}{(T'_d - T''_d)} \frac{L'_d}{(L_d - L'_d)} \frac{M_{df}}{R_f}$$

Model I

A state space model using some assumptions, so that the state space model is in terms of states that can be related to the original states easily. The parameters of this model need to be back-calculated from the standard parameters.

Original Equations in the d-axis

$$\psi_d = L_d i_d + M_{df} i_f + M_{dh} i_h$$

$$\psi_f = M_{df} i_d + L_{ff} i_f + L_{fh} i_h$$

$$\psi_h = M_{dh} i_d + L_{fh} i_f + L_{hh} i_h$$

$$\frac{d\psi_f}{dt} + R_f i_f = v_f$$

$$\frac{d\psi_h}{dt} + R_h i_h = 0$$

$$\frac{d\psi_d}{dt} = -\omega \psi_q - R_a i_d - v_d$$

Alternative d-axis variables

Define New Variables (similar to referring variables on to one side of a transformer)

$$\psi'_f = \psi_f \alpha_f$$

$$\psi'_h = \psi_h \alpha_h$$

$$i'_f = \frac{i_f}{\alpha_f}$$

$$i'_h = \frac{i_h}{\alpha_h}$$

Equations in New Variables

$$\psi_d = L_d i_d + M'_{df} i'_f + M'_{dh} i'_h$$

$$\psi'_f = M'_{df} i_d + L'_{ff} i'_f + L'_{fh} i'_h$$

$$\psi'_h = M'_{dh} i_d + L'_{fh} i'_f + L'_{hh} i'_h$$

$$\frac{d\psi'_f}{dt} + R'_f i'_f = v'_f$$

$$\frac{d\psi'_h}{dt} + R'_h i'_h = 0$$

$$\frac{d\psi_d}{dt} = -\omega\psi_q - R_a i_d - v_d$$

Equations in New Variables

$$M'_{df} = \alpha_f M_{df}, \quad M'_{dh} = \alpha_h M_{dh}$$

$$L'_{ff} = \alpha_f^2 L_{ff}, \quad L'_{fh} = \alpha_h \alpha_f L_{fh}$$

$$R'_f = \alpha_f^2 R_f, \quad v'_f = \alpha_f v_f$$

$$R'_h = \alpha_h^2 R_h \quad L'_{hh} = \alpha_h^2 L_{hh}$$

Simplifications to Reduce Number of Parameters

1. Choose α_h so that $M'_{dh} = M'_{df}$
2. If α_f is the actual turns ratio between stator and field winding, then $M'_{df} = L_d - L_l$, L_l is a leakage inductance.
3. **Assume** $M'_{df} = L'_{fh}$

Back-calculation of Required Parameters

NOTE: Form of the transfer functions is unchanged:

$$\Psi_d(s) = L_d(s) I_d(s) + G'(s) V_f'(s)$$

$$L_d(s) = L_d \frac{(1 + B_N s + A_N s^2)}{(1 + B_D s + A_D s^2)}$$

$$G'(s) = \frac{M_{df}'}{R_f'} \frac{(1 + A_G s)}{(1 + A_D s + B_D s^2)}$$

$$B_N = T'_d + T''_d = \frac{L'_{ff}}{R'_f} + \frac{L'_{hh}}{R'_h} - \frac{{M'_{dh}}^2}{L_d R'_h} - \frac{{M'_{df}}^2}{L_d R'_f}$$

$$A_N = T'_d T''_d = \frac{L'_{ff} L'_{hh}}{R'_f R'_h} - \frac{{L'_{fh}}^2}{R'_f R'_h} - \frac{{M'_{df}}^2 L'_{hh}}{L_d R'_f R'_h} - \frac{{M'_{dh}}^2 L'_{ff}}{L_d R'_f R'_h} + 2\frac{M'_{dh} M'_{df} L'_{fh}}{L_d R'_f R'_h}$$

$$B_D = T'_{do} + T''_{do} = \frac{L'_{ff}}{R'_f} + \frac{L'_{hh}}{R'_h}$$

$$A_D = T'_{do} T''_{do} = \frac{L'_{ff} L'_{hh}}{R'_f R'_h} - \frac{{L'_{fh}}^2}{R'_f R'_h}$$

Thus, one may obtain $L'_{ff}, L'_{hh}, R'_f, R'_h$, given

$$L_d, T'_d, T''_d, T'_{do}, T''_{do}, L_l$$

Parameters for this Model (with assumptions):

$$L_d, L'_{ff}, L'_{hh}, R'_f, R'_h, L_l$$

Parameters from measurement:

$$L_d, T'_d, T''_d, T'_{do}, T''_{do}, L_l$$

R_a is available from measurement.

α_f is not explicitly required if referred voltage v'_f is used in all calculations.

Summary: Model I (d axis)

$$\psi_d = L_d i_d + (L_d - L_l) i'_f + (L_d - L_l) i'_h$$

$$\psi'_f = (L_d - L_l) i_d + L'_{ff} i'_f + (L_d - L_l) i'_h$$

$$\psi'_h = (L_d - L_l) i_d + (L_d - L_l) i'_f + L'_{hh} i'_h$$

$$\frac{d\psi'_f}{dt} + R'_f i'_f = v'_f$$

$$\frac{d\psi'_h}{dt} + R'_h i'_h = 0$$

$$\frac{d\psi_d}{dt} = -\omega\psi_q - R_a i_d - v_d$$

Parameters for this Model (with assumptions):

$$L_d, L'_{ff}, L'_{hh}, R'_f, R'_h, L_l$$

Parameters from measurement:

$$L_d, T'_d, T''_d, T'_{do}, T''_{do}, L_l$$

R_a is available from measurement.

α_f is not explicitly required if referred voltage v'_f is used in all calculations.

Summary: Model I (q axis)

$$\psi_q = L_q i_q + (L_q - L_l) i'_g + (L_q - L_l) i'_k$$

$$\psi'_g = (L_q - L_l) i_q + L'_{gg} i'_g + (L_q - L_l) i'_k$$

$$\psi'_k = (L_q - L_l) i_q + (L_q - L_l) i'_g + L'_{kk} i'_k$$

$$\frac{d\psi'_g}{dt} + R'_g i'_g = 0$$

$$\frac{d\psi'_k}{dt} + R'_k i'_k = 0$$

$$\frac{d\psi_q}{dt} = \omega \psi_d - R_a i_q - v_q$$

Parameters for this Model :

$$L_q, L'_{gg}, L'_{kk}, R'_g, R'_k, L_l$$

Parameters from measurement:

$$L_q, T'_q, T''_q, T'_{qo}, T''_{qo}, L_l$$

R_a is available from measurement.

Model I Equivalent Circuits

Model II

A state space model using some assumptions, so that the state space model is in terms of states that can be related to the original states approximately. The parameters of this model do not need to be back-calculated from the standard parameters.

d-axis variables

Define New Variables (similar to referring variables on to one side of a transformer)

$$\psi_F = \psi_F \alpha_F$$

$$\psi_H = \psi_H \alpha_H$$

$$i_F = \frac{i_F}{\alpha_F}$$

$$i_H = \frac{i_H}{\alpha_H}$$

Equations in New Variables

$$\psi_d = L_d i_d + M'_{df} i_F + M'_{dh} i_H$$

$$\psi_F = M'_{df} i_d + L'_{ff} i_F + L'_{fh} i_H$$

$$\psi_H = M'_{dh} i_d + L'_{fh} i_F + L'_{hh} i_H$$

$$\frac{d\psi_F}{dt} + R'_f i_F = v_F$$

$$\frac{d\psi_H}{dt} + R'_h i_H = 0$$

$$\frac{d\psi_d}{dt} = -\omega\psi_q - R_a i_d - v_d$$

Equations in New Variables

$$M'_{df} = \alpha_F M_{df}, \quad M'_{dh} = \alpha_H M_{dh}$$

$$L'_{ff} = \alpha^2_F L_{ff}, \quad L'_{fh} = \alpha_H \alpha_F L_{fh}$$

$$R'_f = \alpha^2_F R_f, \quad v_F = \alpha_F v_f$$

$$R'_h = \alpha^2_H R_h \quad L'_{hh} = \alpha^2_H L_{hh}$$

Assumptions to Reduce Number of Parameters

1. Choose α_H and α_F so that $M'_{dh} = M'_{df} = L_d$
2. α_F is the not the actual turns ratio between stator and field winding (but approximately equal to it).
3. **Assume** $L_d = L'_{fh}$

Interesting Observations

The assumption made in this model leads us to:

1. $\mathbf{T''_{dc} = T''_{d}}$
2. $T''_{d} = \frac{L'_{hh} - L_d}{R'_h}$
3. $T'_{d} = \frac{L'_{ff} - L_d}{R'_f}$
4. $\frac{1}{L'_d} = \frac{1}{L_d} + \frac{1}{L'_{ff} - L_d}$
5. $\frac{1}{L''_d} = \frac{1}{L'_d} + \frac{1}{L'_{hh} - L_d}$

Equations in New Variables

$$\psi_d = L_d i_d + L_d i_F + L_d i_H$$

$$\psi_F = L_d i_d + L'_{ff} i_F + L_d i_H$$

$$\psi_H = L_d i_d + L_d i_F + L'_{hh} i_H$$

$$\frac{d\psi_F}{dt} + R'_f i_F = v'_f$$

$$\frac{d\psi_H}{dt} + R'_h i_H = 0$$

$$\frac{d\psi_d}{dt} = -\omega \psi_q - R_a i_d - v_d$$

Parameters for this Model :

$$L_d, L'_{ff}, L'_{hh}, R'_f, R'_h$$

Parameters from measurement:

$$L_d, T'_d, T''_d, T'_{do}, T''_{do}$$

R_a is available from measurement.

d-axis Model II

$$\frac{d\psi_H}{dt} = \frac{1}{T''_d}(-\psi_H + \psi_d)$$

$$\frac{d\psi_F}{dt} = \frac{1}{T'_d}(-\psi_F + \psi_d) + \frac{L'_d M'_{df}}{(L_d - L'_d)R'_f T'_d} v'_f$$

$$\psi_d = L''_d i_d + \frac{(L'_d - L''_d)}{L'_d}\psi_H + \frac{(L_d - L'_d)}{L_d}\frac{L''_d}{L'_d}\psi_F$$

$$\frac{d\psi_d}{dt} = -\omega\psi_q - R_a i_d - v_d$$

$$\psi_F = L_d \ i_d + (L_d + \frac{L_d \ L_d'}{L_d - L_d'}) \ i_F + L_d \ i_H$$

$$\psi_H = L_d \ i_d + L_d \ i_F + (L_d + \frac{L_d' \ L_d''}{L_d' - L_d''}) \ i_H$$

q-axis Model II

$$\frac{d\psi_G}{dt} = \frac{1}{T'_q}(-\psi_G + \psi_q)$$

$$\frac{d\psi_K}{dt} = \frac{1}{T''_q}(-\psi_K + \psi_q)$$

$$\psi_q = L''_q i_q + \frac{(L'_q - L''_q)}{L'_q}\psi_K + \frac{(L_q - L'_q)}{L_q}\frac{L''_q}{L'_q}\psi_G$$

$$\frac{d\psi_q}{dt} = \omega\psi_d - R_a i_q - v_q$$

Model II Equivalent Circuits

Per-Unit System

Definition of Base Values

1. V_B = Rated Line-Line Voltage,
2. MVA_B = Rated MVA (three phase)
3. ω_B= Rated (electrical) frequency (rad/s)
4. $I_B = \frac{MVA_B}{V_B}, \omega_{mB} = \frac{2}{P}\omega_B$
5. $T_B = \frac{MVA_B}{\omega_{mB}}, \psi_B = \frac{V_B}{\omega_B}$
6. $Z_B = \frac{V_B}{I_B}, L_B = \frac{Z_B}{\omega_B}$

Model I -PER UNIT

$$\overline{\psi}_d = \overline{x}_d \overline{i}_d + (\overline{x}_d - \overline{x}_l)\overline{i}'_f + (\overline{x}_d - \overline{x}_l)\overline{i}'_h$$

$$\overline{\psi}'_f = (\overline{x}_d - \overline{x}_l)\overline{i}_d + \overline{x}'_{ff}\overline{i}'_f + (\overline{x}_d - \overline{x}_l)\overline{i}'_h$$

$$\overline{\psi}'_h = (\overline{x}_d - \overline{x}_l)\overline{i}_d + (\overline{x}_d - \overline{x}_l)\overline{i}'_f + \overline{x}'_{hh}\overline{i}'_h$$

$$\frac{d\overline{\psi}'_f}{dt} + \omega_B \overline{R}'_f \overline{i}'_f = \omega_B \overline{v}'_f$$

$$\frac{d\overline{\psi}'_h}{dt} + \omega_B \overline{R}'_h \overline{i}'_h = 0$$

$$\frac{d\overline{\psi}_d}{dt} = -\omega\overline{\psi}_q - \omega_B \overline{R}_a \overline{i}_d - \omega_B \overline{v}_d$$

ω and ω_B are in rad/s

Model I - PER UNIT

per-unit line to line voltage under open circuit conditions **in steady state** and when $\omega = \omega_B$

$$\overline{E}_{fd} = \frac{\overline{x}'_{df}}{\overline{R}'_f}\overline{v}'_f = \frac{\overline{x}_d - \overline{x}_l}{\overline{R}'_f}\overline{v}'_f$$

Zero Sequence Equations in pu

(not required for balanced situations)

$$\frac{d\overline{\psi}_o}{dt} = \omega_B \overline{R}_a \overline{i}_o - \omega_B \overline{v}_o$$

Model I -PER UNIT

$$\overline{\psi}_q = \overline{x}_q \overline{i}_q + (\overline{x}_q - \overline{x}_l)\overline{i}'_g + (\overline{x}_q - \overline{x}_l)\overline{i}'_k$$

$$\overline{\psi}'_g = (\overline{x}_q - \overline{x}_l)\overline{i}_q + \overline{x}'_{gg}\overline{i}'_g + (\overline{x}_q - \overline{x}_l)\overline{i}'_k$$

$$\overline{\psi}'_k = (\overline{x}_q - \overline{x}_l)\overline{i}_d + (\overline{x}_q - \overline{x}_l)\overline{i}'_g + \overline{x}'_{kk}\overline{i}'_k$$

$$\frac{d\overline{\psi}'_g}{dt} + \overline{R}'_g \overline{i}'_g = 0$$

$$\frac{d\overline{\psi}'_k}{dt} + \overline{R}'_k \overline{i}'_k = 0$$

$$\frac{d\overline{\psi}_q}{dt} = \omega\overline{\psi}_d - \omega_B \overline{R}_a \overline{i}_q - \omega_B \overline{v}_q$$

ω and ω_B are in rad/s

Model II - PER UNIT

$$\frac{d\overline{\psi}_H}{dt} = \frac{1}{T''_d}(-\overline{\psi}_H + \overline{\psi}_d)$$

$$\frac{d\overline{\psi}_F}{dt} = \frac{1}{T'_d}(-\overline{\psi}_F + \overline{\psi}_d + \frac{\overline{x}'_d}{(\overline{x}_d - \overline{x}'_d)}E_{fd})$$

$$\overline{\psi}_d = \overline{x}''_d\overline{i}_d + \frac{(\overline{x}'_d - \overline{x}''_d)}{\overline{x}'_d}\overline{\psi}_H + \frac{(\overline{x}_d - \overline{x}'_d)}{\overline{x}_d}\frac{\overline{x}''_d}{\overline{x}'_d}\overline{\psi}_F$$

$$\frac{d\overline{\psi}_d}{dt} = -\omega\overline{\psi}_q - \omega_B\overline{R}_a\overline{i}_d - \omega_B\overline{v}_d$$

ω and ω_B are in rad/s

Model II - PER UNIT

$\overline{E}_{fd}$ = line-line open circuit voltage in per-unit **in steady state** and when $\omega = \omega_B$.

Zero Sequence Equations in pu

(not required for balanced situations)

$$\frac{d\overline{\psi}_o}{dt} = \omega_B \overline{R}_a \overline{i}_o - \omega_B \overline{v}_o$$

Model II - PER UNIT

$$\frac{d\overline{\psi}_G}{dt} = \frac{1}{T'_q}(-\overline{\psi}_G + \overline{\psi}_q)$$

$$\frac{d\overline{\psi}_K}{dt} = \frac{1}{T''_q}(-\overline{\psi}_K + \overline{\psi}_q)$$

$$\overline{\psi}_q = \overline{x}''_q \overline{i}_q + \frac{(\overline{x}'_q - \overline{x}''_q)}{\overline{x}'_q}\overline{\psi}_K + \frac{(\overline{x}_q - \overline{x}'_q)}{\overline{x}_q}\frac{\overline{x}''_q}{\overline{x}'_q}\overline{\psi}_G$$

$$\frac{d\overline{\psi}_q}{dt} = \omega\overline{\psi}_d - \omega_B \overline{R}_a \overline{i}_q - \omega_B \overline{v}_q$$

ω and ω_B are in rad/s

Torque Equation in per-unit

$$\frac{2H}{\omega_B}\frac{d\omega}{dt} = T_m - (\overline{\psi}_d \overline{i}_q - \overline{\psi}_q \overline{i}_d)$$

ω is the electrical angular speed in rad/s.

ω_B is the base electrical angular speed in rad/s.

Model which will be used in all future discussions

Model II - PER UNIT

$$\frac{d\psi_H}{dt} = \frac{1}{T''_d}(-\psi_H + \psi_d)$$

$$\frac{d\psi_F}{dt} = \frac{1}{T'_d}\left(-\psi_F + \psi_d + \frac{x'_d}{(x_d - x'_d)}E_{fd}\right)$$

$$\psi_d = x''_d i_d + \frac{(x'_d - x''_d)}{x'_d}\psi_H + \frac{(x_d - x'_d)}{x_d}\frac{x''_d}{x'_d}\psi_F$$

$$\frac{d\psi_d}{dt} = -\omega\psi_q - \omega_B R_a i_d - \omega_B v_d$$

ω and ω_B are in rad/s

Model II - PER UNIT

E_{fd} = line-line open circuit voltage in per-unit **in steady state** and when $\omega = \omega_B$

Zero Sequence Equations in pu

(not required for balanced situations)

$$\frac{d\psi_o}{dt} = \omega_B R_a i_o - \omega_B v_o$$

Model II - PER UNIT

$$\frac{d\psi_G}{dt} = \frac{1}{T_q'}(-\psi_G + \psi_q)$$

$$\frac{d\psi_K}{dt} = \frac{1}{T_q''}(-\psi_K + \psi_q)$$

$$\psi_q = x_q'' i_q + \frac{(x_q' - x_q'')}{x_q'}\psi_K + \frac{(x_q - x_q')}{x_q}\frac{x_q''}{x_q'}\psi_G$$

$$\frac{d\psi_q}{dt} = \omega\psi_d - \omega_B R_a i_q - \omega_B v_q$$

ω and ω_B are in rad/s

Torque Equation in per-unit

$$\frac{2H}{\omega_B}\frac{d\omega}{dt} = T_m - (\psi_d i_q - \psi_q i_d)$$

ω is the electrical angular speed in rad/s.

ω_B is the base electrical angular speed in rad/s.