Synchronous Machine pu Model

Compact Form of Flux Equations

$$\frac{d}{dt}\begin{bmatrix}\psi_d\\\psi_q\\\psi_F\\\psi_H\\\psi_G\\\psi_K\end{bmatrix} = A_1\begin{bmatrix}\psi_d\\\psi_q\\\psi_F\\\psi_H\\\psi_G\\\psi_K\end{bmatrix} + A_2\begin{bmatrix}i_d\\i_q\end{bmatrix}$$

$$+B_1\begin{bmatrix}v_d\\v_q\end{bmatrix} + B_2E_{fd}$$

$$A_3 \begin{bmatrix} \psi_d \\ \psi_q \\ \psi_F \\ \psi_H \\ \psi_G \\ \psi_K \end{bmatrix} = \begin{bmatrix} i_d \\ i_q \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & -\omega & 0 & 0 & 0 & 0 \\ \omega & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{T_d'} & 0 & -\frac{1}{T_d'} & 0 & 0 & 0 \\ \frac{1}{T_d''} & 0 & 0 & -\frac{1}{T_d''} & 0 & 0 \\ 0 & \frac{1}{T_q'} & 0 & 0 & -\frac{1}{T_q'} & 0 \\ 0 & \frac{1}{T_q''} & 0 & 0 & 0 & -\frac{1}{T_q''} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -\omega_B R a & 0 \\ 0 & -\omega_B R a \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -\omega_B & 0 \\ 0 & -\omega_B \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T'_d}\frac{x'_d}{(x_d - x'_d)} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A_3 =$$

$$\begin{bmatrix} \frac{1}{x''_d} & 0 & -\frac{(x_d - x'_d)}{x_d x'_d} & -\frac{(x'_d - x''_d)}{x'_d x''_d} & 0 & 0 \\ 0 & \frac{1}{x''_q} & 0 & 0 & -\frac{(x_q - x'_q)}{x_q x'_q} & -\frac{(x'_q - x''_q)}{x'_q x''_q} \end{bmatrix}$$

q-axis Model - per-unit

$$\frac{d\psi_G}{dt} = \frac{1}{T'_q}(-\psi_G + \psi_q)$$

$$\frac{d\psi_K}{dt} = \frac{1}{T''_q}(-\psi_K + \psi_q)$$

$$\psi_q = x''_q i_q + \frac{(x'_q - x''_q)}{x'_q}\psi_K + \frac{(x_q - x'_q)}{x_q}\frac{x''_q}{x'_q}\psi_G$$

$$\frac{d\psi_q}{dt} = \omega\psi_d - \omega_B R_a i_q - \omega_B v_q$$

d-axis Model - in pu (assuming $T''_{dc} = T''_d$)

$$\frac{d\psi_H}{dt} = \frac{1}{T''_d}(-\psi_H + \psi_d)$$

$$\frac{d\psi_F}{dt} = \frac{1}{T'_d}\left(-\psi_F + \psi_d + \frac{x'_d}{(x_d - x'_d)}E_{fd}\right)$$

$$\psi_d = x''_d i_d + \frac{(x'_d - x''_d)}{x'_d}\psi_H + \frac{(x_d - x'_d)}{x_d}\frac{x''_d}{x'_d}\psi_F$$

$$\frac{d\psi_d}{dt} = -\omega\psi_q - \omega_B R_a i_d - \omega_B v_d$$

$$E_{fd} = \frac{\omega_B M_{df}}{R_f} \frac{v_f}{V_{BASE}}$$

Zero Sequence Equations in pu

(not required for balanced situations)

$$\frac{d\psi_o}{dt} = \omega_B R_a i_o - \omega_B v_o$$

Torque Equation in per-unit

$$\frac{2H}{\omega_B}\frac{d\omega}{dt} = T_m - (\psi_d i_q - \psi_q i_d)$$

ω is the electrical angular speed in rad/s.

ω_B is the base electrical angular speed in rad/s.

Approximate Models

If studying slow electromechanical transients, while operating near the nominal speed, use

$$0 = -\omega_B \psi_q - \omega_B R_a i_d - \omega_B v_d$$

$$0 = \omega_B \psi_d - \omega_B R_a i_q - \omega_B v_q$$

i.e., **set**

$$\frac{d\psi_d}{dt} = 0 \qquad \frac{d\psi_d}{dt} = 0 \qquad \omega \approx \omega_B$$

Approximate Models: ‘2.1’ Model

If the effect of one q-axis damper winding is neglected (i.e., the $'k'$ winding), set

$$R_k = \infty \Rightarrow \; T_q'' = 0$$

Therefore,

$$T_q''\frac{d\psi_K}{dt} = 0$$

2.1 : Two windings on d axis, One on q-axis

q-axis equations: 2.1 model

$$\frac{d\psi_G}{dt} = \frac{1}{T'_q}(-\psi_G + \psi_q)$$

$$\psi_q = x'_q i_q + \frac{(x_q - x'_q)}{x_q}\psi_G$$

$$\frac{d\psi_q}{dt} = \omega\psi_d - \omega_B R_a i_q - \omega_B v_q$$

The last equation can also be made an algebraic equation when studying slow transients

d-axis equations 1.1 model

$$\frac{d\psi_F}{dt} = \frac{1}{T'_d}(-\psi_F + \psi_d + \frac{x'_d}{(x_d - x'_d)}E_{fd})$$

$$\psi_d = x'_d i_d + \frac{(x_d - x'_d)}{x_d}\psi_F$$

$$\frac{d\psi_d}{dt} = -\omega\psi_q - \omega_B R_a i_d - \omega_B v_d$$

The last equation can also be made an algebraic equation when studying slow transients

q-axis equations 1.0 model

$$\psi_q = x_q i_q$$

$$\frac{d\psi_q}{dt} = \omega \psi_d - \omega_B R_a i_q - \omega_B v_q$$

The last equation can also be made an algebraic equation when studying slow transients

Classical Model: 0.0 model

$$R_f \to 0 \Rightarrow T'_d \to \infty$$

$$\frac{d\psi_F}{dt} = 0$$

$$\psi_d = x'_d i_d + \frac{(x_d - x'_d)}{x_d}\psi_F$$

$$\frac{d\psi_d}{dt} = -\omega\psi_q - \omega_B R_a i_d - \omega_B v_d$$

Classical Model

$$\psi_d = x'_d i_d + E'$$

$$0 = -\omega_B \psi_q - \omega_B R_a i_d - \omega_B v_d$$

$$\psi_q = x_q i_q$$

$$0 = \omega_B \psi_d - \omega_B R_a i_q - \omega_B v_q$$

Further Approximations : $R_a = 0,\ x_q = x'_d$