AVR+Synchronous Machine pu Model

Torque Equation in per-unit

$$\frac{2H}{\omega_B}\frac{d\omega}{dt} = T_m - (\psi_d i_q - \psi_q i_d)$$

ω is the electrical angular speed in rad/s.

ω_B is the base electrical angular speed in rad/s.

Compact Form of Flux Equations

$$\frac{d}{dt}\begin{bmatrix}\psi_d\\ \psi_q\\ \psi_F\\ \psi_H\\ \psi_G\\ \psi_K\end{bmatrix} = A_1\begin{bmatrix}\psi_d\\ \psi_q\\ \psi_F\\ \psi_H\\ \psi_G\\ \psi_K\end{bmatrix} + A_2\begin{bmatrix}i_d\\ i_q\end{bmatrix}$$

$$+B_1\begin{bmatrix}v_d\\ v_q\end{bmatrix} + B_2E_{fd}$$

$$A_3 \begin{bmatrix} \psi_d \\ \psi_q \\ \psi_F \\ \psi_H \\ \psi_G \\ \psi_K \end{bmatrix} = \begin{bmatrix} i_d \\ i_q \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & -\omega & 0 & 0 & 0 & 0 \\ \omega & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{T'_d} & 0 & -\frac{1}{T'_d} & 0 & 0 & 0 \\ \frac{1}{T''_d} & 0 & 0 & -\frac{1}{T''_d} & 0 & 0 \\ 0 & \frac{1}{T'_q} & 0 & 0 & -\frac{1}{T'_q} & 0 \\ 0 & \frac{1}{T''_q} & 0 & 0 & 0 & -\frac{1}{T''_q} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -\omega_B R_a & 0 \\ 0 & -\omega_B R_a \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -\omega_B & 0 \\ 0 & -\omega_B \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T'_d}\frac{x'_d}{(x_d - x'_d)} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A_3 =$$

$$\begin{bmatrix} \frac{1}{x''_d} & 0 & -\frac{(x_d - x'_d)}{x_d x'_d} & -\frac{(x'_d - x''_d)}{x'_d x''_d} & 0 & 0 \\ 0 & \frac{1}{x''_q} & 0 & 0 & -\frac{(x_q - x'_q)}{x_q x'_q} & -\frac{(x'_q - x''_q)}{x'_q x''_q} \end{bmatrix}$$

Model of interconnection:

$$\begin{bmatrix} L & 0 & 0 \\ 0 & L & 0 \\ 0 & 0 & L \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \left(\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} - \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix}\right.$$

Model of interconnection (d-q pu form)

$$\frac{d}{dt}\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{\omega_B}{x}\left(\begin{bmatrix} v_d \\ v_q \end{bmatrix} - \begin{bmatrix} E_d \\ E_q \end{bmatrix}\right)$$

$$x = \frac{\omega_B L}{Z_{base}}$$

Infinite Bus

$$E_d = -E\sin\delta \quad E_q = E\cos\delta$$

$$\frac{d\delta}{dt} = \omega - \omega_o$$

$$E = 1, \omega_o = \omega_B$$

If studying **slow** electromechanical transients, while operating near the nominal speed, use

$$0 = -\omega_B \psi_q - \omega_B R_a i_d - \omega_B v_d$$

$$0 = \omega_B \psi_d - \omega_B R_a i_q - \omega_B v_q$$

i.e., **in the stator flux differential equations set**

$$\frac{d\psi_d}{dt} = 0 \qquad \frac{d\psi_q}{dt} = 0 \qquad \omega \approx \omega_B$$

Model of interconnection (d-q pu form)

$$\frac{d}{dt}\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{\omega_B}{x}\left(\begin{bmatrix} v_d \\ v_q \end{bmatrix} - \begin{bmatrix} E_d \\ E_q \end{bmatrix}\right)$$

$$x = \frac{\omega_B L}{Z_{base}}$$

A logical extension of the approximation ...

set

$$\frac{di_d}{dt} = 0 \quad \frac{di_q}{dt} = 0 \quad \omega \approx \omega_B$$

Therefore the differential equation for the inter-connection becomes an algebriac equation.

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & -\omega_B \\ \omega_B & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{\omega_B}{x} \left(\begin{bmatrix} v_d \\ v_q \end{bmatrix} - \begin{bmatrix} E_d \\ E_q \end{bmatrix} \right)$$

Torque Equation in per-unit

$$\frac{2H}{\omega_B}\frac{d\omega}{dt} = T_m - (\psi_d i_q - \psi_q i_d)$$

ω is the electrical angular speed in rad/s.

ω_B is the base electrical angular speed in rad/s.

Rotor Flux Equations

$$\frac{d}{dt}\begin{bmatrix}\psi_F\\ \psi_H\\ \psi_G\\ \psi_K\end{bmatrix} = A_1'\begin{bmatrix}\psi_F\\ \psi_H\\ \psi_G\\ \psi_K\end{bmatrix} + A_1''\begin{bmatrix}\psi_d\\ \psi_q\end{bmatrix}$$
$$+B_2'E_{fd}$$

Note: Stator flux (ψ_d, ψ_q) differential equations become algebraic equations, due to the neglect of fast transients.

$$A_1' = \begin{bmatrix} -\frac{1}{T_d'} & 0 & 0 & 0 \\ 0 & -\frac{1}{T_d''} & 0 & 0 \\ 0 & 0 & -\frac{1}{T_q'} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_q''} \end{bmatrix}$$

$$A_1'' = \begin{bmatrix} \frac{1}{T_d'} & 0 \\ \frac{1}{T_d''} & 0 \\ 0 & \frac{1}{T_q'} \\ 0 & \frac{1}{T_q''} \end{bmatrix}$$

$$B_2' = \begin{bmatrix} \frac{1}{T_d'} \frac{x_d'}{(x_d - x_d')} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A_3 \begin{bmatrix} \psi_d \\ \psi_q \\ \psi_F \\ \psi_H \\ \psi_G \\ \psi_K \end{bmatrix} = \begin{bmatrix} i_d \\ i_q \end{bmatrix}$$

$$A_3 = \begin{bmatrix} \frac{1}{x_d''} & 0 & -\frac{(x_d - x_d')}{x_d x_d'} & -\frac{(x_d' - x_d'')}{x_d' x_d''} & 0 & 0 \\ 0 & \frac{1}{x_q''} & 0 & 0 & -\frac{(x_q - x_q')}{x_q x_q'} & -\frac{(x_q' - x_q'')}{x_q' x_q''} \end{bmatrix}$$

Infinite Bus

$$E_d = -E \sin \delta \quad E_q = E \cos \delta$$

$$\frac{d\delta}{dt} = \omega - \omega_o$$

$$E = 1, \omega_o = \omega_B$$

Simple Static Exciter Model

$$\frac{dX_E}{dt} = \frac{1}{T_A}\left(-X_E + k_A\left(V_{ref} - V\right)\right)$$

$$V = \sqrt{v_d^2 + v_q^2}$$

States, Inputs

1. States: $\delta, \omega, \psi_F, \psi_H, \psi_G, \psi_K$
2. Number of differential equations: 6
3. Other variables: $\psi_d, \psi_q, i_d, i_q, v_d, v_q$
4. Algebriac Equations : 6
5. Inputs: T_m, E_{fd}

$\psi_d, \psi_q, i_d, i_q, v_d, v_q$ can be obtained in terms of the states using the 6 LINEAR algebraic Equations

$$0 = -\omega_B \psi_q - \omega_B R_a i_d - \omega_B v_d$$

$$0 = \omega_B \psi_d - \omega_B R_a i_q - \omega_B v_q$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & -\omega_B \\ \omega_B & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{\omega_B}{x} \left(\begin{bmatrix} v_d \\ v_q \end{bmatrix} - \begin{bmatrix} E_d \\ E_q \end{bmatrix} \right)$$

$$A_3 \begin{bmatrix} \psi_d & \psi_q & \psi_F & \psi_H & \psi_G & \psi_K \end{bmatrix}^T = \begin{bmatrix} i_d \\ i_q \end{bmatrix}$$