Linearized Analysis

Equilibrium Condition Calculation

- Infinite bus voltage is GIVEN - $E\angle 0$

$$E_{an} = \sqrt{\frac{2}{3}} E \sin(\omega_o t)$$

$$E_{bn} = \sqrt{\frac{2}{3}} E \sin(\omega_o t - \frac{2\pi}{3})$$

$$E_{cn} = \sqrt{\frac{2}{3}} E \sin(\omega_o t - \frac{4\pi}{3})$$

- Steady state generator Power output and Voltage Magnitude are specified: P, V.

- Transmission Line reactance is specified: X
- Phase angle of generator terminal voltage:

$$\theta_t = \sin^{-1} \frac{PX}{VE}$$

- Generator bus voltage is $V\angle\theta$:

$$v_{an} = \sqrt{\frac{2}{3}} V \sin(\omega_o t + \theta_t)$$

$$v_{bn} = \sqrt{\frac{2}{3}} V \sin(\omega_o t - \frac{2\pi}{3} + \theta_t)$$

$$v_{cn} = \sqrt{\frac{2}{3}} V \sin(\omega_o t - \frac{4\pi}{3} + \theta_t)$$

- Line Current = Generator Current is $I\angle\phi$:

$$i_a = \sqrt{\frac{2}{3}} I \sin(\omega_o t + \phi)$$

$$i_b = \sqrt{\frac{2}{3}} I \sin(\omega_o t - \frac{2\pi}{3} + \phi)$$

$$i_c = \sqrt{\frac{2}{3}} I \sin(\omega_o t - \frac{4\pi}{3} + \phi)$$

$$I = \left\| \frac{V\angle\theta_t - E\angle 0}{jX} \right\|$$

$$\phi = \angle \frac{V\angle\theta_t - E\angle 0}{jX}$$

$$E_d = -E\sin\delta \quad E_q = E\cos\delta$$

$$v_d = V\sin(\theta_t - \delta) \quad v_q = V\cos(\theta_t - \delta)$$

$$i_d = I\sin(\phi - \delta) \quad i_q = I\cos(\phi - \delta)$$

$$(E_q + jE_d)e^{j\delta} = E\angle 0$$

$$(v_q + jv_d)e^{j\delta} = V\angle\theta_t$$

$$(i_q + ji_d)e^{j\delta} = I\angle\phi$$

If R_a is neglected and $\omega_o = \omega_B$ then in steady state,

$$(E_{fd} + (x_d - x_q)i_d)e^{j\delta}$$
$$= (v_q + jv_d)e^{j\delta} + jx_q(i_q + ji_d)e^{j\delta}$$
$$= V\angle\theta + jx_q I\angle\phi$$

Therefore δ is

$$\delta = \angle(V\angle\theta_t + j\ x_q\ I\angle\phi)$$

From $(i_q + ji_d)e^{j\delta} = I\angle\phi$, obtain $i_d,\ \ i_q$

- $(E_{fd} + (x_d - x_q)i_d)$ is known, therefore E_{fd} is obtained

- From E_{fd} obtain

$$V_{ref} = V + \frac{E_{fd}}{k_A}$$

- Initial speed $\omega = \omega_o$

- Compute equilibrium values of all d axis fluxes from:

$$0 = \psi_d - v_q$$

$$0 = -\psi_H + \psi_d$$

$$0 = -\psi_F + \psi_d + \frac{x'_d}{x_d - x'_d} E_{fd}$$

- Similarly for the q axis, in steady state,

$$0 = -\psi_q - v_d$$

$$0 = -\psi_G + \psi_q$$

$$0 = -\psi_K + \psi_q$$

Linearization

$$\frac{2H}{\omega_B}\frac{d\Delta\omega}{dt} = \Delta T_m - \Delta(\psi_d i_q - \psi_q i_d)$$

$$\Delta(\psi_d i_q - \psi_q i_d) =$$

$$\psi_{do}\Delta i_q - \psi_{qo}\Delta i_d + i_{qo}\Delta\psi_d - i_{do}\Delta\psi_q$$

$$\frac{d\Delta\delta}{dt} = \Delta\omega$$

Linearization: Flux Equations

$$\frac{d}{dt}\begin{bmatrix}\Delta\psi_d\\ \Delta\psi_q\\ \Delta\psi_F\\ \Delta\psi_H\\ \Delta\psi_G\\ \Delta\psi_K\end{bmatrix} = A_{10}\begin{bmatrix}\Delta\psi_d\\ \Delta\psi_q\\ \Delta\psi_F\\ \Delta\psi_H\\ \Delta\psi_G\\ \Delta\psi_K\end{bmatrix} + A_2\begin{bmatrix}\Delta i_d\\ \Delta i_q\end{bmatrix}$$

$$+B_1\begin{bmatrix}\Delta v_d\\ \Delta v_q\end{bmatrix} + B_2\Delta E_{fd} + A_{11}\Delta\omega$$

$$A_3 \begin{bmatrix} \Delta\psi_d \\ \Delta\psi_q \\ \Delta\psi_F \\ \Delta\psi_H \\ \Delta\psi_G \\ \Delta\psi_K \end{bmatrix} = \begin{bmatrix} \Delta i_d \\ \Delta i_q \end{bmatrix}$$

$$A_{10} = \begin{bmatrix} 0 & -\omega & 0 & 0 & 0 & 0 \\ \omega & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{T'_d} & 0 & -\frac{1}{T'_d} & 0 & 0 & 0 \\ \frac{1}{T''_d} & 0 & 0 & -\frac{1}{T''_d} & 0 & 0 \\ 0 & \frac{1}{T'_q} & 0 & 0 & -\frac{1}{T'_q} & 0 \\ 0 & \frac{1}{T''_q} & 0 & 0 & -\frac{1}{T''_q} & \end{bmatrix}$$

$$A_{11} = \begin{bmatrix} -\psi_{qo} \\ \psi_{do} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -\omega_B R a & 0 \\ 0 & -\omega_B R a \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -\omega_B & 0 \\ 0 & -\omega_B \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T'_d}\frac{x'_d}{(x_d - x'_d)} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A_3 =$$

$$\begin{bmatrix} \frac{1}{x''_d} & 0 & -\frac{(x_d - x'_d)}{x_d x'_d} & -\frac{(x'_d - x''_d)}{x'_d x''_d} & 0 & 0 \\ 0 & \frac{1}{x''_q} & 0 & 0 & -\frac{(x_q - x'_q)}{x_q x'_q} & -\frac{(x'_q - x''_q)}{x'_q x''_q} \end{bmatrix}$$

Model of Interconnection

$$\frac{d}{dt}\begin{bmatrix}\Delta i_d \\ \Delta i_q\end{bmatrix} = \begin{bmatrix}0 & -\omega_o \\ \omega_o & 0\end{bmatrix}\begin{bmatrix}\Delta i_d \\ \Delta i_q\end{bmatrix}$$

$$+\frac{\omega_B}{X}\left(\begin{bmatrix}\Delta v_d \\ \Delta v_q\end{bmatrix} - \begin{bmatrix}\Delta E_d \\ \Delta E_q\end{bmatrix}\right) + \begin{bmatrix}-i_{qo} \\ i_{do}\end{bmatrix}\Delta\omega$$

$$\frac{\omega_B}{X}\begin{bmatrix}\Delta v_d \\ \Delta v_q\end{bmatrix} = \frac{d}{dt}\begin{bmatrix}\Delta i_d \\ \Delta i_q\end{bmatrix} - \begin{bmatrix}0 & -\omega_o \\ \omega_o & 0\end{bmatrix}\begin{bmatrix}\Delta i_d \\ \Delta i_q\end{bmatrix}$$

$$+\frac{\omega_B}{X}\begin{bmatrix}\Delta E_d \\ \Delta E_q\end{bmatrix} - \begin{bmatrix}-i_{qo} \\ i_{do}\end{bmatrix}\Delta\omega$$

- Substitute for Δi_d and Δi_q in terms of the fluxes.
- Substitute for the derivatives of the fluxes. Δv_d and Δv_q are now algebraically related to the other states.

Static Exciter Model

$$\frac{d\Delta E_{fd}}{dt} = \frac{1}{T_A}\left(-\Delta E_{fd} + k_A \left(\Delta V_{ref} - \Delta V\right)\right)$$

$$V = \sqrt{v_d^2 + v_q^2}$$

$$\Delta V = \left[\frac{v_{do}}{V_o} \quad \frac{v_{qo}}{V_o}\right] \begin{bmatrix} \Delta v_d \\ \Delta v_q \end{bmatrix}$$

$$\Delta E_d = -E \cos \delta_o \Delta \delta$$

$$\Delta E_q = -E \sin \delta_o \Delta \delta$$

Finally!

$$\frac{d\Delta x}{dt} = A\Delta x + B\Delta u$$

$$\Delta x = [\Delta\delta \;\; \Delta\omega \;\; \Delta\psi_F \;\; \Delta\psi_H \;\; \Delta\psi_G \;\; \Delta\psi_K \;\; \Delta E_{fd}]^T$$

$$\Delta u = [\Delta T_m \;\;\; \Delta V_{ref}]^T$$

States: 7