Synchronous Machine pu Model

q-axis Model - per-unit

$$\frac{d\psi_G}{dt} = \frac{1}{T'_q}(-\psi_G + \psi_q)$$

$$\frac{d\psi_K}{dt} = \frac{1}{T''_q}(-\psi_K + \psi_q)$$

$$\psi_q = x''_q i_q + \frac{(x'_q - x''_q)}{x'_q}\psi_K + \frac{(x_q - x'_q)}{x_q}\frac{x''_q}{x'_q}\psi_G$$

$$\frac{d\psi_q}{dt} = \omega\psi_d - \omega_B R_a i_q - \omega_B v_q$$

d-axis Model - in pu (assuming $T''_{dc} = T''_d$)

$$\frac{d\psi_H}{dt} = \frac{1}{T''_d}(-\psi_H + \psi_d)$$

$$\frac{d\psi_F}{dt} = \frac{1}{T'_d}(-\psi_F + \psi_d + \frac{x'_d}{(x_d - x'_d)}E_{fd})$$

$$\psi_d = x''_d i_d + \frac{(x'_d - x''_d)}{x'_d}\psi_H + \frac{(x_d - x'_d)}{x_d}\frac{x''_d}{x'_d}\psi_F$$

$$\frac{d\psi_d}{dt} = -\omega\psi_q - \omega_B R_a i_d - \omega_B v_d$$

$$E_{fd} = \frac{\omega_B M_{df}}{R_f} \frac{v_f}{V_{BASE}}$$

Zero Sequence Equations in pu

(not required for balanced situations)

$$\frac{d\psi_o}{dt} = \omega_B R_a i_o - \omega_B v_o$$

Torque Equation in per-unit

$$\frac{2H}{\omega_B}\frac{d\omega}{dt} = T_m - (\psi_d i_q - \psi_q i_d)$$

ω is the electrical angular speed in rad/s.

ω_B is the base electrical angular speed in rad/s.

Approximate Models

If studying slow electromechanical transients, while operating near the nominal speed, use

$$0 = -\omega_B \psi_q - \omega_B R_a i_d - \omega_B v_d$$

$$0 = \omega_B \psi_d - \omega_B R_a i_q - \omega_B v_q$$

i.e., **set**

$$\frac{d\psi_d}{dt} = 0 \qquad \frac{d\psi_d}{dt} = 0 \qquad \omega \approx \omega_B$$

$$[C_{P1}] =$$

$$\sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta_1 & \sin\theta_1 & \sqrt{\frac{1}{2}} \\ \cos(\theta_1 - 2\pi/3) & \sin(\theta_1 - 2\pi/3) & \sqrt{\frac{1}{2}} \\ \cos(\theta_1 + 2\pi/3) & \sin(\theta_1 + 2\pi/3) & \sqrt{\frac{1}{2}} \end{bmatrix}$$

$$\theta_1 = \omega_1 t = \omega_o t + \delta_1$$

Alternative Transformation

$$[C_K] =$$

$$\sqrt{\frac{2}{3}} \begin{bmatrix} \cos \omega_o t & \sin \omega_o t & \sqrt{\frac{1}{2}} \\ \cos(\omega_o t - 2\pi/3) & \sin(\omega_o t - 2\pi/3) & \sqrt{\frac{1}{2}} \\ \cos(\omega_o t + 2\pi/3) & \sin(\omega_o t + 2\pi/3) & \sqrt{\frac{1}{2}} \end{bmatrix}$$

Alternative Transformation

$$\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} = [C_{P1}] \begin{bmatrix} f_{d1} \\ f_{q1} \\ f_o \end{bmatrix} = [C_K] \begin{bmatrix} f_D \\ f_Q \\ f_o \end{bmatrix}$$

$$(f_Q + jf_D) = (f_{q1} + jf_{d1})e^{j\delta_1}$$