Two Machine System pu Model

Differential Equations: Network Transients Neglected

$$\frac{d\psi_G}{dt} = \frac{1}{T'_q}(-\psi_G + \psi_q)$$

$$\frac{d\psi_K}{dt} = \frac{1}{T''_q}(-\psi_K + \psi_q)$$

$$\frac{d\psi_H}{dt} = \frac{1}{T''_d}(-\psi_H + \psi_d)$$

$$\frac{d\psi_F}{dt} = \frac{1}{T'_d}(-\psi_F + \psi_d + \frac{x'_d}{(x_d - x'_d)}E_{fd})$$

Simple Static Exciter + AVR Model

$$\frac{dX_E}{dt} = \frac{1}{T_A}\left(-X_E + k_A \left(V_{ref} - V\right)\right)$$

$$V = \sqrt{v_d^2 + v_q^2} = \sqrt{v_D^2 + v_Q^2}$$

$E_{fd} = X_E$ except that it is clipped if limits are exceeded

Simple Turbine- Governor Model

$$\frac{\Delta P_m(s)}{\Delta \omega(s)} = K \frac{1+2s}{1+6s}$$

$$P_m = P_{mo} + \Delta P_m$$

$$\Delta \omega = \omega_{REF} - \omega$$

P_m is limited.

Mechanical Equations in per-unit

$$\frac{d\delta}{dt} = \omega - \omega_o$$

$$\frac{2H}{\omega_B}\frac{d\omega}{dt} = T_m - (\psi_d i_q - \psi_q i_d)$$

$$\approx P_m - (\psi_d i_q - \psi_q i_d)$$

Mechanical Equations in per-unit

ω is the electrical angular speed in rad/s.

ω_B is the base electrical angular speed in rad/s.

$\theta = \omega_o t + \delta$, is the rotor position.

In this study, let us take, $\omega_o = \omega_B$

Algebraic Equations (Both Machines)

$$\psi_d = x''_d i_d + \frac{(x'_d - x''_d)}{x'_d}\psi_H + \frac{(x_d - x'_d)}{x_d}\frac{x''_d}{x'_d}\psi_F$$

$$0 = -\omega_B \psi_q - \omega_B R_a i_d - \omega_B v_d$$

$$\psi_q = x''_q i_q + \frac{(x'_q - x''_q)}{x'_q}\psi_K + \frac{(x_q - x'_q)}{x_q}\frac{x''_q}{x'_q}\psi_G$$

$$0 = \omega_B \psi_d - \omega_B R_a i_q - \omega_B v_q$$

D-Q variables

$$[C_K] =$$

$$\sqrt{\frac{2}{3}} \begin{bmatrix} \cos \omega_o t & \sin \omega_o t & \sqrt{\frac{1}{2}} \\ \cos(\omega_o t - 2\pi/3) & \sin(\omega_o t - 2\pi/3) & \sqrt{\frac{1}{2}} \\ \cos(\omega_o t + 2\pi/3) & \sin(\omega_o t + 2\pi/3) & \sqrt{\frac{1}{2}} \end{bmatrix}$$

D-Q variables

$$\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} = [C_{P1}] \begin{bmatrix} f_{d1} \\ f_{q1} \\ f_o \end{bmatrix} = [C_K] \begin{bmatrix} f_{D1} \\ f_{Q1} \\ f_o \end{bmatrix}$$

D-Q variables

$$\begin{bmatrix} \cos\delta_1 & \sin\delta_1 \\ -\sin\delta_1 & \cos\delta_1 \end{bmatrix} \begin{bmatrix} f_{d1} \\ f_{q1} \end{bmatrix} = \begin{bmatrix} f_{D1} \\ f_{Q1} \end{bmatrix}$$

$$(f_{Q1} + jf_{D1}) = (f_{q1} + jf_{d1})e^{j\delta_1}$$

For Machine 2:

$$(f_{Q2} + jf_{D2}) = (f_{q2} + jf_{d2})e^{j\delta_2}$$

Algebraic Equations (Both Machines): Assume $x''_d = x''_q$

$$\psi_D = x''_d i_D + \mathcal{F}_1(\psi_H, \psi_G, \psi_K, \psi_F, \delta)$$

$$0 = -\omega_B \psi_Q - \omega_B R_a i_D - \omega_B v_D$$

$$\psi_Q = x''_d i_Q + \mathcal{F}_2(\psi_H, \psi_G, \psi_K, \psi_F, \delta)$$

$$0 = \omega_B \psi_D - \omega_B R_a i_Q - \omega_B v_Q$$

Algebraic Equations: Network + Load: Transients Neglected

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{r\omega_B}{x} & -\omega_B \\ \omega_B & -\frac{r\omega_B}{x} \end{bmatrix} \begin{bmatrix} i_{lD} \\ i_{lQ} \end{bmatrix} + \frac{\omega_B}{x} \left(\begin{bmatrix} v_{D1} \\ v_{Q1} \end{bmatrix} - \begin{bmatrix} v_{D2} \\ v_{Q2} \end{bmatrix} \right)$$

Algebraic Equations: Network + Load: Transients Neglected

$$v_{Q1} = R_{L1}(i_{Q1} - i_{lQ1})$$

$$v_{D1} = R_{L1}(i_{D1} - i_{lD1})$$

$$v_{Q2} = R_{L2}(i_{Q2} + i_{lQ2})$$

$$v_{D2} = R_{L2}(i_{D2} + i_{lD2})$$

Differential Equations / States: 16

$$\delta_1, \omega_1, \psi_{G1}, \psi_{H1}, \psi_{K1}, \psi_{F1}, X_{E1}, X_{G1}$$

$$\delta_2, \omega_2, \psi_{G2}, \psi_{H2}, \psi_{K2}, \psi_{F2}, X_{E2}, X_{G2}$$

Algebraic Equations / Variables: 14

$$\psi_{D1}, \psi_{Q1}, \psi_{D2}, \psi_{Q2}, i_{D1}, i_{Q1}, i_{D2},$$

$$i_{Q2}, v_{D1}, v_{Q1}, v_{D2}, v_{Q2}, i_{lD}, i_{lQ}$$